نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ..... کلاس .....

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$$f(x)=\begin{cases}\cot\frac{\pi x}{۱۲} & x\le ۱\\ \sqrt{x^۲+۱} & x>۱\end{cases}$$

$fof(\frac{۲}{۳})=?$ $\quad f(\frac{۲}{۳})\Rightarrow \cot\frac{\pi}{۶}\frac{۲}{۱۲}=\cot\frac{\pi}{۶}=\sqrt{۳}$

$f(\sqrt{۳})\Rightarrow \sqrt{(\sqrt{۳})^۲+۱}=\sqrt{۴}=۲$ $\quad fof(\frac{۲}{۳})=\boxed{۲}$

۲

الف) $f(\frac{۱}{x})=\sqrt{\frac{۲x-۱}{x^۲}}$

$g(x)=۲\cos^۲x \longrightarrow g(\frac{\pi}{۳})=۲(\frac{\pi}{۳})^۲=\frac{۱}{۲}$ $\quad \frac{۱}{x}=\frac{۱}{۲}\ x=۲\ f(۲)=۶$

$\frac{\sqrt{۲\times۲-۱}}{۲}=\frac{\sqrt{۳}}{۲}$ $\quad fog(\frac{\pi}{۳})=\boxed{\frac{\sqrt{۳}}{۲}}$

ب) $f(x)=[x]$

$g(x)=\frac{x}{۱-x}\longrightarrow g(\sqrt{۲})=\frac{\sqrt{۲}}{۱-\sqrt{۲}}\times\frac{۱+\sqrt{۲}}{۱+\sqrt{۲}}=\frac{\sqrt{۲}+۲}{-۱}=-۲-\sqrt{۲}$

$\simeq -۴$ $\quad f(-۲-\sqrt{۲})=[-۲-\sqrt{۲}]$

$fog(\sqrt{۲})\neq\boxed{-۴}$

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$f(x)=\sin x$

$g(x)=x\sqrt{۱-x^۲}$

$gof(\frac{\pi}{۴})\longrightarrow f(\frac{\pi}{۴})=\sin\frac{\pi}{۴}=\frac{\sqrt{۲}}{۲}$ $\quad g(\frac{\sqrt{۲}}{۲})=\frac{\sqrt{۲}}{۲}\sqrt{۱-(\frac{\sqrt{۲}}{۲})^۲}=\frac{\sqrt{۲}}{۲}\times\frac{۱}{\sqrt{۲}}=\frac{۱}{۲}$

$gof(\frac{\pi}{۴})=\boxed{\frac{۱}{۲}}$

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$f(x)=\{(۲,۵),(۷,۵),(۸,۱۲),(۱۰,۱۲)\}$ $\quad g(x)=\{(۲,۷),(۲,۴),(۷,۸),(۸,۱۰)\}$

الف) $fog(x)$ $\quad ۴\xrightarrow{g(x)}۷\xrightarrow{f(x)}۵$ $\quad ۲\longrightarrow۴\longrightarrow۵$ $\quad ۷\longrightarrow۸\longrightarrow۱۲$ $\quad ۸\longrightarrow۱۰\longrightarrow۱۲$

$=[(۴,۵),(۲,۵),(۷,۱۲),(۸,۱۲)]$

ب) $gof(x)=۴\xrightarrow{f(x)}۵$ $\quad ۷\longrightarrow۵$ $\quad ۸\longrightarrow۱۲$ $\quad ۱۰\longrightarrow۱۲$ ∅

ج) $fof(x)=۴\xrightarrow{f(x)}۵$ $\quad ۷\longrightarrow۵$ $\quad ۸\longrightarrow۱۲$ $\quad ۱۰\longrightarrow۱۲$ ∅

د) $gog(x)=۴\xrightarrow{g(x)}۷\xrightarrow{g(x)}۸$ $\quad ۴\longrightarrow۴\longrightarrow۷$ $\quad ۷\longrightarrow۸\longrightarrow۱۰$ $\quad ۸\longrightarrow۱۰$ ✗

$=[(۴,۸),(۲,۷),(۷,۱۰)]$

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$f=\{(۲,۱),(۴,۲),(۶,۵),(۱,۷)\}$ $\quad g(x)=\{(۱,۲),(۳,۱),(a,۳),(b,۱)\}$

$fog\Rightarrow ۱\xrightarrow{g(x)}۲\xrightarrow{f(x)}۱$ $\quad ۳\longrightarrow۱\xrightarrow{f(x)}۷$ $\quad a\longrightarrow۳\longrightarrow۲$ $\quad b\longrightarrow۱\longrightarrow۷$

$(a,۲)\in(۴,۲)\longrightarrow a=۴$

$gof\Rightarrow ۲\xrightarrow{f(x)}۱\xrightarrow{g(x)}۲$ $\quad ۳\longrightarrow۲$ ✗ $\quad ۴\longrightarrow۵\longrightarrow۱$ $\quad ۱\longrightarrow۷$ ✗

$(b,۱)=(۵,۱)\longrightarrow b=۵$

$(a,b)=\boxed{(۴,۵)}$

$f(f(x)) \Rightarrow a(ax+b)+b = 4x+3 \quad \underbrace{a^2}_{4}x + \underbrace{b(a+1)}_{3} = 4x+3$ → $a = 2$ یا -2 ; $a=2 \to 2x+1$; $a=-2 \to -2x-3$ ۶

$g(2x+3) = 3x-1 \quad \xrightarrow{2x+3=t} \quad g(t) = \frac{3t-13}{2}$

$gof(-1)$ → $2(-1)+1=-1 \quad g(-1) = \frac{3(-1)-13}{2} = \frac{-16}{2} = -8$

→ $-2(-1)-3=-1 \quad g(-1) = \frac{3(-1)-13}{2} = -8$ } $gof(-1) = \boxed{-8}$

$f(x) = \sqrt{x+|x|}$

$g(x) = \frac{1}{x^2-4x} \longrightarrow D_{gof} = \{x \in D_f \,,\, f(x) \in D_g\}$ ۷

$D_{gof} = \{\mathbb{R} \,,\, \sqrt{x+|x|} \neq [0,4]\} \Rightarrow D_{gof} = \mathbb{R} - [0,4]$

$f(x) = \sqrt{1-x^2} \quad g(x) = \sqrt{x}$

$(f+g)of \longrightarrow [0,1] \cup \underbrace{(\sqrt{1-x^2})}_{-1 \le x \le 1} = \boxed{[-1,1]}$

↓ ۸

$\underbrace{\sqrt{1-x^2}}_{-1 \le x \le 1} + \underbrace{\sqrt{x}}_{x \ge 0} \xrightarrow{\cap} [0,1]$

الف) $f\left(\frac{3x+1}{x-2}\right) = 4x+5 \quad \frac{3x+1}{x-2} = t \quad tx-2t = 3x+1 \quad x(t-3) = 2t+1$

$x = \frac{2t+1}{t-3} \quad f(t) = 4 \times \frac{2t+1}{t-3} + 5 \to \frac{8t+4+5t-15}{t-3} = \frac{13t-11}{t-3}$

$f(x) = \frac{13x-11}{x-3}$ ۹

ب) $f(x+\frac{1}{x}) = x^3 + \frac{1}{x^3} \quad x+\frac{1}{x} = t \quad t^3 = x^3 + \frac{1}{x^3} + 3(x+\frac{1}{x})$

$f(t) = t^3 - 3x - \frac{3}{x} \Rightarrow f(t) = t^3 - 3t \quad f(x) = x^3 - 3x$

$g(1) = 0 \quad g(2\sqrt{2}) = 0$

برخورد با محورها → $\underbrace{g}_{1 \text{ یا } 2\sqrt{2}}(f(x)) = 0 \quad \underbrace{f(x)=1}$ یا $\underbrace{f(x) = 2\sqrt{2}}$

$x\sqrt{x} = 1 \quad \boxed{x=1} \quad x\sqrt{x} = 2\sqrt{2} \quad \boxed{x=2}$ ۱۰

$x_2 - x_1 = 2 - 1 = \boxed{1}$

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