(۱۹)

نام و نام خانوادگی ..شایان ..... پاسخنامه تشریحی تکلیف شماره ..۹.... کلاس ...یازدهم تجربی B......

$f(x)=\begin{cases}\cot\frac{\pi x}{4} & x\le 1\\ \sqrt{x^2+1} & x>1\end{cases}$ $f\circ f(\frac{2}{3})=?$ ① $\cot\frac{\pi\times\frac{2}{3}}{4}=\cot\frac{\pi}{6}\to\frac{\pi}{6}=30\to\cot 30=\sqrt{3}$ ② $\sqrt{3}\ge 1\to\sqrt{(\sqrt{3})^2+1}=\sqrt{4}=2$ (۳)	۱
الف) $f\circ g(\frac{\pi}{3})\Rightarrow f(g(\frac{\pi}{3}))\Rightarrow g(\frac{\pi}{3})=2\cos^2(\frac{\pi}{3})\Rightarrow 2\times(\frac{1}{2})^2=\frac{1}{2}$ $\Rightarrow f(\frac{1}{2})=\sqrt{\frac{4-1}{4}}=\sqrt{\frac{3}{4}}$ ب) $f\circ g(\sqrt{2})\Rightarrow g(\sqrt{2})=\frac{\sqrt{2}}{1-\sqrt{2}}\times\frac{1+\sqrt{2}}{1+\sqrt{2}}\to\frac{\sqrt{2}(1+\sqrt{2})}{1-2}=\frac{\sqrt{2}+2}{-1}=-(\sqrt{2}+2)$ $f(-\sqrt{2}-2)=[-\sqrt{2}-2]=[-3,...]=-4$ $f\circ g(\sqrt{2})=-4$ (۴)	۲
$f(x)=\sin x$ $g(x)=x\sqrt{1-x^2}$ $g\circ f(\frac{\pi}{4})$ ... $\frac{\pi}{4}=\frac{\sqrt{2}}{2}=\sin 45$ $\Rightarrow\frac{\sqrt{2}}{2}\sqrt{1-\frac{1}{2}}\to\frac{\sqrt{2}}{2}\times\sqrt{\frac{1}{2}}\Rightarrow\sqrt{\frac{2}{8}}=\sqrt{\frac{1}{4}}=\frac{1}{2}$ $g\circ f(\frac{\pi}{4})=\frac{1}{2}$ (۴)	۳
الف) $f\circ g$: $4\xrightarrow{g(x)}6\xrightarrow{f(x)}5$; $2\to4\to5$; $6\to8\to12$; $8\to10\to12$ $f\circ g=\{(4,5),(2,5),(6,12),(8,12)\}$ ج) $f\circ f$: $4\xrightarrow{f(x)}5\xrightarrow{f(x)}\times$; $6\to5\to\times$; $8\to12\to\times$; $10\to12\to\times$ $f\circ f=\emptyset$ ب) $g\circ f$: $4\xrightarrow{f(x)}5\xrightarrow{g(x)}\times$; $8\to12\to\times$; $6\to5\to\times$; $10\to12\to\times$ $g\circ f=\emptyset$ د) $g\circ g$: $4\xrightarrow{g(x)}6\xrightarrow{g(x)}8$; $2\to4\to6$; $6\to8\to10$; $\to10$ $g\circ g=\{(4,8),(2,6),(6,10)\}$ (۴)	۴
$f=\{(2,1),(4,2),(4,5),(1,7)\}$ $g=\{(1,2),(4,1),(a,2),(b,1)\}$ $(4,2)\in f\circ g$ $(4,1)\in g\circ f$ (a,b) دوتایی $f\circ g\Rightarrow 1\xrightarrow{g(x)}2\xrightarrow{f(x)}1$ $2\xrightarrow{g(x)}1\xrightarrow{f(x)}1$ $a\xrightarrow{g(x)}2\xrightarrow{f(x)}2\to(4,2)\to a=4$ $b\xrightarrow{g(x)}1\xrightarrow{f(x)}7$ $g(\underbrace{f(x)}_{b})=1\Rightarrow f(4)=5=b$ $a=4,\ b=5=(4,5)$ (۴)	۵

۶

$f \circ f = f(ax+b) \to a(ax+b)+b \Rightarrow f \circ f = a^2x+ab+b = 4x+6 \to$

$f(x) = -2x-2$ or $f(x) = 2x+1$

$x = -1 \to f(-1) = -1 \qquad g(2x+2) = 4x-1$

$g(-1) = ? \to 2x-2 = -1 \to x = -\frac{1}{2} \quad g(-1) = -8$ (3)

$g \circ f(-1) = -8$

۷

$f(x) = \sqrt{x+|x|} \to x+|x| \geq 0 \to x \geq 0 \to D_f = [0, +\infty)$

$g(x) = \frac{1}{x^2-4x} > 0 \to x(x-4) > 0 \to x = 0, 4 \Rightarrow \mathbb{R} - \{0, 4\}$ (3)

$g \circ f = \{x \in D_f, f(x) \in D_g\}$

$f(0) = \sqrt{0} = 0 \notin D_g$

$f(x) = \sqrt{2x} = 2 \to x = 2 \Rightarrow D_{g \circ f} = (0,2) \cup (2, +\infty)$

۸

$(f+g) \circ f = \sqrt{x} + \sqrt{1-x^2} = \sqrt{\sqrt{1-x^2}} + \sqrt{1+1+x^2}$ (1)

$\sqrt{1-x^2} + |x| \to 1-x^2 \geq 0 \to 1 \geq x^2 \to -1 \leq x \leq 1$

$D_g = \mathbb{R} \cap D_f = -1 \leq x \leq 1 \to 0 \leq x \leq 1$

$D_{(f+g) \circ f} = [-1, 1] \to 0 \leq \sqrt{1-x^2} \leq 1 \to x^2 \geq 0$ ✓

۹

الف) $f\left(\frac{2x+1}{x-2}\right) = 6x+5 \to \frac{2x+1}{x-2} = t \Rightarrow tx-2t = 2x+1 \Rightarrow x(t-2) = 2t+1$

$x = \frac{2t+1}{t-2}$

بنابراین $f(x) = 4\left(\frac{2t+1}{t-2}\right)+5 \Rightarrow \frac{8t+4+5t-10}{t-2} = \frac{13t-11}{t-2}$

$f(x) = \frac{13x-11}{x-2}$

(3)

ب) $f\left(x+\frac{1}{x}\right) = x^3+\frac{1}{x^3} \to x+\frac{1}{x} = t \to \left(x+\frac{1}{x}\right)^3 = x^3+3x+\frac{1}{x^3}+\frac{3}{x} = 0$

$\left(x+\frac{1}{x}\right)^3 + 3\left(x+\frac{1}{x}\right) \quad x+\frac{1}{x} = t \quad t^3+3t = f(t) \to f(x) = x^3+3x$

۱۰

$g(1) = 0$

$g(2\sqrt{2}) = 0$

$f(x) = x\sqrt{x} \Rightarrow x\sqrt{x} = 1 \Rightarrow x = 1 = \sqrt{1}$

$x\sqrt{x} = 2\sqrt{2} \to x = 2$

اختلاف طول $= 2-1 = 1$ (3)