

$f(x) = \begin{cases} \cot \frac{\pi x}{4} & x \leq 1 \\ \sqrt{x^2 + 1} & x > 1 \end{cases}$ $f \circ f\left(\frac{1}{\sqrt{2}}\right) = ?$	۱) $\cot \frac{\pi \times \frac{1}{\sqrt{2}}}{4} = \cot \frac{\pi}{4} \rightarrow \frac{\pi}{4} = \pi \rightarrow \cot \pi = \infty$ ۲) $\sqrt{2} \geq 1 \rightarrow \sqrt{(\sqrt{2})^2 + 1} = \sqrt{3}$ ۱
الف) $f \circ g\left(\frac{\pi}{4}\right) = f\left(g\left(\frac{\pi}{4}\right)\right) \rightarrow g\left(\frac{\pi}{4}\right) = r \cos^2\left(\frac{\pi}{4}\right) \Rightarrow r \times \left(\frac{1}{2}\right) = \frac{1}{2}$ $\Rightarrow f\left(\frac{1}{2}\right) = \sqrt{\frac{1}{4} - 1} = \sqrt{\frac{3}{4}}$ ب) $f \circ g(\sqrt{2}) \Rightarrow g(\sqrt{2}) = \frac{\sqrt{2}}{1 - \sqrt{2}} \times \frac{1 + \sqrt{2}}{1 + \sqrt{2}} \rightarrow \frac{\sqrt{2}(1 + \sqrt{2})}{1 - 2} = \frac{\sqrt{2} + 2}{-1} = -(\sqrt{2} + 2)$ $f(-(\sqrt{2} + 2)) = [-\sqrt{2} - 2] = [-2, -\infty] = -\infty$ $f \circ g(\sqrt{2}) = -\infty$	۲
$f(x) = \sin x$ $g(x) = x\sqrt{1 - x^2}$ $g \circ f\left(\frac{\pi}{6}\right) = \frac{\pi}{6} \sqrt{1 - \left(\frac{1}{2}\right)^2} = \frac{\pi}{6} \sqrt{\frac{3}{4}} = \frac{\pi}{6} \times \frac{\sqrt{3}}{2} = \frac{\pi\sqrt{3}}{12}$ $g \circ f\left(\frac{\pi}{6}\right) = \frac{\pi}{6}$	۳
الف) $f \circ g$ $\begin{matrix} \mathbb{R} & \xrightarrow{g} & \mathbb{R} & \xrightarrow{f} & \mathbb{R} \\ 1 & \xrightarrow{g} & 2 & \xrightarrow{f} & 4 \\ 2 & \xrightarrow{g} & 4 & \xrightarrow{f} & 16 \\ 4 & \xrightarrow{g} & 16 & \xrightarrow{f} & 256 \end{matrix}$ $f \circ g = \{(1, 4), (2, 16), (4, 256), (16, 65536)\}$ ب) $g \circ f$ $\begin{matrix} \mathbb{R} & \xrightarrow{f} & \mathbb{R} & \xrightarrow{g} & \mathbb{R} \\ 1 & \xrightarrow{f} & 1 & \xrightarrow{g} & 1 \\ 2 & \xrightarrow{f} & 4 & \xrightarrow{g} & 4 \\ 4 & \xrightarrow{f} & 16 & \xrightarrow{g} & 16 \end{matrix}$ $g \circ f = \{(1, 1), (2, 4), (4, 16)\}$	۴
$f = \{(1, 1), (2, 2), (3, 3), (4, 4)\}$ $g = \{(1, 1), (2, 2), (3, 3), (4, 4)\}$ $(1, 1) \in f \circ g$ $(2, 2) \in g \circ f$ (a, b) روی	$f \circ g = \{(1, 1), (2, 2), (3, 3), (4, 4)\}$ $g \circ f = \{(1, 1), (2, 2), (3, 3), (4, 4)\}$ $f(g(x)) = 1 \rightarrow f(x) = 1 \rightarrow x = 1$ $g(f(x)) = 1 \rightarrow g(x) = 1 \rightarrow x = 1$ $a = 1, b = 1 \Rightarrow (1, 1)$ ۵

$f \circ f = f(ax+b) \rightarrow a(ax+b)+b \Rightarrow f \circ f = a^2x+ab+bx+ba \Rightarrow$ $f(x) = -x-1 \text{ or } f(x) = x+1$ $x = -1 \rightarrow f(-1) = -1 \quad g(x+1) = x-1$ $g(-1) = ? \rightarrow x-1 = -1 \rightarrow x = 0 \quad g(-1) = -1$ $g \circ f(-1) = -1$	6
$f(x) = \sqrt{x+ x } \rightarrow 0 \leq x+ x \rightarrow x \geq 0 \rightarrow Df = [0, +\infty)$ $g(x) = \frac{1}{x^2-4x}, 0 \rightarrow x(x-4) > 0 \rightarrow x < 0 \text{ or } x > 4 \Rightarrow Dg = \mathbb{R} - \{0, 4\}$ $g \circ f = \{x \in Df, f(x) \in Dg\}$ $\text{مثلاً } f(0) = \sqrt{0} = 0 \rightarrow g(0) \text{ غير معرف}$ $f(4) = \sqrt{4+4} = \sqrt{8} \rightarrow g(\sqrt{8}) \text{ غير معرف}$ $\Rightarrow Dg \circ f = (0, 4) \cup (4, +\infty)$	7
$(f+g) \circ f = \sqrt{x} + \sqrt{1-x^2} = \sqrt{1-x^2} + \sqrt{1+1+x^2} \Rightarrow$ $\sqrt{1-x^2} + x \rightarrow 1-x^2 \geq 0 \rightarrow 1 \geq x^2 \Rightarrow -1 \leq x \leq 1$ $D(f+g) \circ f = [-1, 1]$	8
$\text{جاء } f\left(\frac{x+1}{x-1}\right) = t \Rightarrow \frac{x+1}{x-1} = t \Rightarrow tx - 1 = x + 1 \Rightarrow x(t-1) = 2 \Rightarrow x = \frac{2}{t-1}$ $\text{نستبدل } f(x) = t \left(\frac{t+1}{t-1} \right) + 2 \Rightarrow \frac{1t + 2(t-1)}{t-1} = \frac{3t-2}{t-1}$ $\frac{3t-2}{t-1} = \frac{1 \cdot \frac{2}{t-1} + 1}{\frac{2}{t-1} - 1} = \frac{\frac{2}{t-1} + 1}{\frac{2-t}{t-1}} = \frac{2+t-1}{2-t} = \frac{t+1}{2-t}$ $\Rightarrow f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2} \Rightarrow x + \frac{1}{x} = t \Rightarrow \left(x + \frac{1}{x}\right)^2 = t^2 \Rightarrow x^2 + \frac{1}{x^2} + 2 = t^2 \Rightarrow x^2 + \frac{1}{x^2} = t^2 - 2$ $\left(x + \frac{1}{x}\right)^2 + 2 \left(x + \frac{1}{x}\right) = t^2 + 2t = f(t) \Rightarrow f(x) = x^2 + 2x$	9
$g(1) = 0$ $g(\sqrt{x}) > 0$ $f(x) = x\sqrt{x} \Rightarrow x\sqrt{x} = 1 \Rightarrow x^2 = 1 \Rightarrow x = 1$ $x\sqrt{x} = \sqrt{x} \Rightarrow x = 1$	10