

$$f \circ f\left(\frac{\pi}{4}\right) \rightarrow f\left(\frac{\pi}{4}\right) = \cot \frac{\pi}{4} = \sqrt{3} \rightarrow f(\sqrt{3}) = \sqrt{3+1} = \boxed{2}$$

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$$\text{الف } f \circ g\left(\frac{\pi}{\sqrt{2}}\right) \rightarrow g\left(\frac{\pi}{\sqrt{2}}\right) = 2 \cos^2 \frac{\pi}{\sqrt{2}} = \frac{1}{\sqrt{2}} \rightarrow f\left(\frac{1}{\sqrt{2}}\right) = \sqrt{\frac{1}{\sqrt{2}}} = \boxed{\frac{\sqrt{2}}{2}}$$

$$\text{ب } f \circ g(\sqrt{2}) \rightarrow g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}} = -\sqrt{2}-2 \rightarrow f(-\sqrt{2}-2) = [-2-\sqrt{2}] = \boxed{-4}$$

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$$g \circ f\left(\frac{\pi}{\sqrt{2}}\right) \rightarrow f\left(\frac{\pi}{\sqrt{2}}\right) = \sin \frac{\pi}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2}} \rightarrow g\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \frac{\sqrt{2}}{\sqrt{2}} \times \sqrt{\frac{1}{\sqrt{2}}} = \boxed{\frac{1}{\sqrt{2}}}$$

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$$f \circ g = \{(f, a)(2, a)(4, 1)(1, 1)\}$$

$$g \circ f = \emptyset$$

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$$(f, 2) \in f \circ g \rightarrow f(x) = 2 \rightarrow x = 2 \rightarrow g(x) = 2 \rightarrow x = f \rightarrow a = f$$

$$(f, 1) \in g \circ f \rightarrow g(x) = 1 \rightarrow x = 2, b \rightarrow b = a$$

$$(a, b) = \boxed{(f, a)}$$

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$$f(x) = ax + b$$

$$f(f(x)) = a^2x + ab + b \quad f(x) = \begin{cases} a=1, b=1 \\ a=-1, b=-1 \end{cases} \rightarrow -1x - 1$$

$$g(1x+1) = 1x-1$$

$$g(x) = a'x + b' \rightarrow 1a'x + 1a' + b' \rightarrow a' = \frac{1}{1}x, b' = -\frac{1}{1}$$

$$g \circ f(-1) \rightarrow g(-1) = -\frac{1}{1} - \frac{1}{1} = \boxed{-2}$$

$$f(x) = \sqrt{x+1} \rightarrow D_f = \mathbb{R} \rightarrow \text{if } x \leq 0 \rightarrow f(x) = 0$$

$$g(x) = \frac{1}{x^2 - f(x)} \rightarrow D_g = \mathbb{R} - \{0, 1\} \rightarrow \mathbb{R}_f \neq 0, 1 \rightarrow \boxed{D = (0, 1) \cup (1, \infty) - \{1\}}$$

$$\left. \begin{aligned} f(x) &= \sqrt{1-x^2} \rightarrow D_f = [-1, 1] \\ g(x) &= \sqrt{x} \rightarrow D_g = [0, \infty) \end{aligned} \right\} \cap = [0, 1]$$

$$(f+g)(x) \rightarrow \sqrt{1-x^2} + \sqrt{x} \rightarrow \boxed{D_f = [0, 1]}$$

$$f\left(\frac{rx+1}{x-r}\right) = f(x) + \omega \rightarrow \frac{rx+1}{x-r} = t \rightarrow x = \frac{rt+1}{t-r} \rightarrow f(x) = f\left(\frac{rt+1}{t-r}\right) + \omega = \frac{1^rt-1}{t-r}$$

$$f(x) = \boxed{\frac{1^rx-1}{x-r}}$$

$$f\left(x + \frac{1}{x}\right) = x^r + \frac{1}{x^r} \rightarrow x + \frac{1}{x} = t \rightarrow t^r = x^r + \frac{1}{x^r} = \underbrace{x^r + \frac{1}{x^r}}_{t^r} = \boxed{f(x) = x^r - \frac{1}{x^r}}$$

$$\left. \begin{aligned} g(\sqrt{r}) = 0 &\rightarrow f(x) = r\sqrt{r} \rightarrow r\sqrt{r} = x\sqrt{x} \rightarrow x = r \\ g(1) = 0 &\rightarrow f(x) = 1 \rightarrow 1 = x\sqrt{x} \rightarrow x = 1 \end{aligned} \right\} r=1 \quad \boxed{1}$$