نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ۹ کلاس یازدهم ریاضی

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$$f(x)=\begin{cases}\cos+\frac{12x}{4} & x\le 1\\ \sqrt{x^2+1} & x>1\end{cases}$$

$$f\circ f\left(\frac{1}{4}\right)=f\left(f\left(\frac{1}{4}\right)\right)=f\left(\sqrt{3}\right)=2$$

$$f\left(\frac{1}{4}\right)=\cos+12\times\frac{\frac{1}{4}}{4}=\cos+\frac{12}{16}=\cos+30^\circ=\sqrt{3}$$

$$f\left(\sqrt{3}\right)=\sqrt{(\sqrt{3})^2+1}=2$$

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الف) $f\left(\frac{1}{x}\right)=\sqrt{\frac{2x-1}{x^2}}\xrightarrow{\frac{1}{x}=t} f(t)=\sqrt{\frac{2(\frac{1}{t})-1}{(\frac{1}{t})^2}}=\sqrt{t^2\left(\frac{2}{t}-1\right)}=\sqrt{2-t^2}$

$\sqrt{t(2-t)}\rightarrow f(x)=\sqrt{x(2-x)}$

$g\left(\frac{\pi}{4}\right)=2\cos^2\left(\frac{\pi}{4}\right)=2\times\left(\frac{1}{2}\right)^2=\frac{1}{2}$

$f\circ g\left(\frac{\pi}{4}\right)=f\left(g\left(\frac{\pi}{4}\right)\right)=f\left(\frac{1}{2}\right)=\sqrt{\frac{1}{2}\left(2-\frac{1}{2}\right)}=\sqrt{\frac{3}{4}}=\frac{\sqrt{3}}{2}$

ب) $g(\sqrt{2})=\frac{(\sqrt{2})(1+\sqrt{2})}{(1-\sqrt{2})(1+\sqrt{2})}=\frac{\sqrt{2}+2}{-1}=-\sqrt{2}-2$

$f(-\sqrt{2}-2)=[-\sqrt{2}-2]=[-3.\_]=-3$

$f(g(\sqrt{2}))=-3$

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$f\left(\frac{\pi}{4}\right)=\sin\frac{\pi}{4}=\frac{\sqrt{2}}{2}$

$g\left(\frac{\sqrt{2}}{2}\right)=\frac{\sqrt{2}}{2}\sqrt{1-\left(\frac{\sqrt{2}}{2}\right)^2}=\frac{\sqrt{2}}{2}\times\sqrt{\frac{1}{2}}=\frac{\sqrt{2}}{2}\times\frac{1}{\sqrt{2}}=\frac{1}{2}$

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$f(x)=\{(3,5)(4,5)(8,12)(10,12)\}$

$g(x)=\{(3,4)(4,8)(8,10)(10,12)\}$

$f\circ g(x)=\{(3,5)(4,5)(8,12)\}$

$4\rightarrow 8\rightarrow 12$, $8\rightarrow 10\rightarrow 12$

$g\circ f(x)=\emptyset$ ($4\xrightarrow{f}5\xrightarrow{g}\times$, $4\xrightarrow{f}5\xrightarrow{g}\times$, $10\xrightarrow{f}12\xrightarrow{g}\times$)

$f\circ f(x)=\emptyset$ ($4\xrightarrow{f}5\xrightarrow{f}\times$, $4\xrightarrow{f}5\xrightarrow{f}\times$, $10\xrightarrow{f}12\xrightarrow{f}\times$)

$g\circ g(x)=\{(3,8)(4,10)\}$ ($2\rightarrow 4\rightarrow 4$, $4\rightarrow 8\rightarrow 10$, $8\rightarrow 10\rightarrow\times$)

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$f=\{(2,1)(3,2)(4,5)(1,7)\}$

$g=\{(1,2)(3,1)(5,4)(b,1)\}$

$f\circ g(4)=2\Rightarrow x=4\xrightarrow{g(x)}1\text{ or }3\longrightarrow 2$

$a\rightarrow 4\rightarrow 2\checkmark\rightarrow a=4$

$b\rightarrow 1\rightarrow 7\times$

$g\circ f(4)=\not\exists\Rightarrow x=4\xrightarrow{f(x)}5\longrightarrow 1$

$(b,1)=(5,1)\longrightarrow b=5$

$(a,b)=(4,5)$

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$f(x)=ax+b \rightarrow f(f(x)) = f(ax+b) = a(ax+b)+b = a^2x+ab+b = 4x+3$

$\rightarrow a^2=4 \rightarrow a=\pm 2$

$a=2 \rightarrow 3b=3 \rightarrow b=1 \rightarrow f(x)=2x+1$

$a=-2 \rightarrow -b=3 \rightarrow b=-3 \rightarrow f(x)=-2x-3$

$g(2x+3)=3x-2 \rightarrow 2x+3=t \rightarrow x=\frac{t-3}{2} \rightarrow g(t)=3\left(\frac{t-3}{2}\right)-2=\frac{3t-9}{2}-2=\frac{3t-13}{2}$

$\rightarrow g(x)=\frac{3x-13}{2}$

$g(f(-1)) = g(-2+1) = g(-1) = \frac{-3-13}{2} = -8$

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$f(x)=\sqrt{x+|x|}$ → R^-: 0 ✓ ، 0: 0 ✓ ، R^+: $2x$ ✓

$D_f = R$

$gof = \frac{1}{(f(x))^2-4f(x)}$

$\rightarrow f(x)(f(x)-4)\neq 0$ → $f(x)\neq 0 \rightarrow R^-, 0$ ✗ ؛ $f(x)\neq 4 \rightarrow \sqrt{2x}=4 \rightarrow 2x=16 \rightarrow x=8$ ✗

$D_{gof} = (0,8)\cup(8,+\infty)$

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$(f+g)of = f(f(x)) + g(f(x))$

$f(f(x)) = \sqrt{1-(f(x))^2} = \sqrt{1-(\sqrt{1-x^2})^2} = \sqrt{1-(1-x^2)} = \sqrt{x^2} = |x|$

$g(f(x)) = \sqrt{f(x)} = \sqrt{\sqrt{1-x^2}} = (1-x^2)^{\frac{1}{4}} \rightarrow \sqrt{1-x^2}\geq 0 \rightarrow 1-x^2 \geq 0 \rightarrow -1\leq x\leq 1$

$f(x)=\sqrt{1-x^2}$ ، $1 \geq x^2 \rightarrow -1\leq x\leq 1$

$\rightarrow \cap = -1\leq x\leq 1$

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الف) $f\left(\frac{3x+1}{x-2}\right) = 4x+5 \rightarrow \frac{3x+1}{x-2}=t \rightarrow \frac{3t+1}{t-2} \rightarrow x=\frac{2t+1}{t-3}$

$f(t) = 4\left(\frac{2t+1}{t-3}\right)+5 = \frac{8t+4+5t-15}{t-3} = \frac{13t-11}{t-3} \rightarrow f(x)=\frac{13x-11}{x-3}$

ب) $f\left(x+\frac{1}{x}\right) = x^3+\frac{1}{x^3} \rightarrow x+\frac{1}{x}=t \rightarrow \left(x+\frac{1}{x}\right)^3 = t^3 \rightarrow x^3+\frac{1}{x^3}+3x+\frac{3}{x} = t^3$ ، $3\left(x+\frac{1}{x}\right) = 3t$

$x^3+\frac{1}{x^3} = t^3-3t \rightarrow f(t)=t^3-3t \rightarrow f(x)=x^3-3x$

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$g(f(x))$ → اگر $f(x)$ با 1 و $2\sqrt{2}$ برابر باشد حاصل $g(x)$ صفر است

$f(x)=1 \rightarrow x\sqrt{x}=1 \rightarrow x=1$

$f(x)=2\sqrt{2} \rightarrow x\sqrt{x}=2\sqrt{2} \rightarrow x=2$

$2-1=1$

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