

$$f(x) = \begin{cases} \cot \frac{\pi}{4} x & x \leq 1 \\ \sqrt{x^2 + 1} & x > 1 \end{cases}$$

$$f\left(\frac{1}{2}\right) = \frac{\pi \cdot \frac{1}{2}}{4} = \frac{\pi}{8} \rightarrow \cot \frac{\pi}{4} = \sqrt{3}$$

$$f(\sqrt{3}) = \sqrt{(\sqrt{3})^2 + 1} = 2$$

$$f \circ f\left(\frac{1}{2}\right) = 2$$

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$$f(x) = [x] \rightarrow [-3, 4] = -3$$

$$g(x) = \frac{x}{1-x} = \frac{\sqrt{3}}{1-\sqrt{3}} = \frac{\sqrt{3}+1}{1-3} = -\frac{1+\sqrt{3}}{2}$$

$$f \circ g(\sqrt{3}) = -4$$

$$f\left(\frac{1}{x}\right) = \sqrt{\frac{2x-1}{x^2+1}} \rightarrow \frac{2x-1}{x^2+1} = \frac{1}{4}$$

$$g(x) = 2 \cos^2 x \rightarrow 2 \cos^2 \frac{\pi}{6} = \frac{1}{2}$$

$$f \circ g\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} \rightarrow \frac{2-1}{4} = \frac{\sqrt{3}}{2}$$

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$$f(x) = \sin x \rightarrow \sin\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

$$g(x) = x \sqrt{1-x^2} \rightarrow \frac{\sqrt{3}}{2} \sqrt{1-\frac{3}{4}} = \frac{\sqrt{3}}{2} \times \frac{\sqrt{1}}{2} = \frac{\sqrt{3}}{4} = \frac{1}{4}$$

$$g \circ f\left(\frac{\pi}{6}\right) = \frac{1}{4}$$

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$$f \circ g(x) = \{(e, d), (c, d), (y, a), (h, a)\}$$

$$g \circ f(x) = \emptyset$$

(۱) الف

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$$g \circ f(x) = \{(e, h), (c, y), (y, a), (h, a)\}$$

$$f \circ f(x) = \emptyset$$

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$$f \circ g = \{(c, a), (c, c), (e, d), (h, a)\}$$

$$g(x) = \{(a, c), (c, h), (a, c), (b, a)\}$$

$$(e, c) \in f \circ g$$

$$f \circ g \begin{cases} 1 \rightarrow c \rightarrow 1 \\ c \rightarrow 1 \rightarrow v \Rightarrow a = c \\ a \rightarrow c \rightarrow c \\ b \rightarrow 1 \rightarrow v \end{cases}$$

$$g \circ f \begin{cases} c \rightarrow 1 \rightarrow c \\ c \rightarrow c \rightarrow x \\ e \rightarrow d \rightarrow 1 \\ 1 \rightarrow v \rightarrow x \end{cases}$$

$$(a, b) = (c, d)$$

(۲) الف

$$f(f(x)) = f(x+2) \rightarrow a(ax+b)+b = a^2x+ab+b \quad a^2 = 5 \rightarrow a = \sqrt{5} \text{ or } a = -\sqrt{5}$$

$$g(2x+3) = 2x-2 \rightarrow a(2x+3)+b = 2ax+3a+b = 2x-2 \rightarrow a = \frac{1}{2}, b = -\frac{13}{2}$$

$$F_n = 2n+1, g_n = \frac{1}{2}n - \frac{13}{2}$$

$$F(-1) = -2+1 = -1, g(-1) = -\frac{1}{2} - \frac{13}{2} = -\frac{14}{2} = -7$$

$$g \circ f(-1) = -8$$

$$f(x) = \sqrt{x+|x|}, g(x) = \frac{1}{x^2-5x}$$

$$D_{g \circ f} = \{x \in D_f : f(x) \in D_g\} \rightarrow \left\{ \begin{array}{l} x \in \mathbb{R} \\ \sqrt{x+|x|} \in \mathbb{R} - \{0, 5\} \end{array} \right\} \rightarrow D_{g \circ f} = (0, +\infty) - \{1\}$$

$$D_f = x+|x| \geq 0 \rightarrow \mathbb{R}$$

$$D_g = x^2-5x \neq 0 \rightarrow x(x-5) \neq 0 \rightarrow \left\{ \begin{array}{l} x \neq 0 \\ x \neq 5 \end{array} \right\} \rightarrow x+|x| \neq 0 \rightarrow x \neq (-\infty, 0]$$

$$f(x) = \sqrt{1-x^2}, g(x) = \sqrt{x}$$

$$D(f+g) \circ f = \{x \in D_f : f(x) \in D_{f+g}\} \rightarrow \{x \in [-1, 1] : x \in [-1, 1]\} \rightarrow D = [-1, 1]$$

$$D_{f+g} = D_f \cap D_g = [-1, 1] \cap [0, +\infty) = [0, 1]$$

$$D_g = x \geq 0, D_f = [-1, 1]$$

$$f\left(\frac{x+1}{x-2}\right) = 5x+4$$

$$\frac{5x+1}{x-2} = t \rightarrow x = \frac{2t+1}{t-5}$$

$$f(t) = 5\left(\frac{2t+1}{t-5}\right) + 4 = \frac{10t+5}{t-5} + 4$$

$$f(x) = \frac{10x+5}{x-5} + 4$$

$$f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2}$$

$$x + \frac{1}{x} = t \rightarrow \left(x + \frac{1}{x}\right)^2 = t^2 \rightarrow x^2 + \frac{1}{x^2} = t^2 - 2$$

$$f(t) = t^2 - 2$$

$$f(x) = x^2 - 2x$$

$$f(x) = x\sqrt{x}$$

$$g \circ f(x) \rightarrow g(f(x)) = 1 \rightarrow f(x) = 1 \rightarrow x\sqrt{x} = 1 \rightarrow x = 1$$

$$g(1) = 0 \rightarrow g(2\sqrt{2}) = 0 \rightarrow f(x) = 2\sqrt{2} \rightarrow x\sqrt{x} = 2\sqrt{2} \rightarrow x = 2$$

$$\text{اختلاف اعداد متتالية} = x_2 - x_1 = 2 - 1 = 1$$