نام و نام خانوادگی: رزا همتی — تکلیف شماره: ۹ — کلاس یازدهم گروه B دفتر تجربی سه‌شنبه‌ها

۱-

$$f(x)=\begin{cases}\cot\frac{\pi}{4}x & x\le 1\\ \sqrt{x^2+1} & x>1\end{cases}$$

$f(\frac{2}{3}) = \dfrac{\pi\frac{2}{3}}{4} = \frac{\pi}{6} \to \cot\frac{\pi}{6} = \sqrt{3}$

$f(\sqrt{3}) = \sqrt{(\sqrt{3})^2+1} = 2$

$fof(\frac{2}{3}) = \boxed{2}$

۲- الف)

$f(\frac{1}{x}) = \sqrt{\dfrac{2x-1}{x^2}}$

$g(x)=2\cos^2 x \to 2\cos^2\frac{\pi}{3} = \frac{1}{2}$

$fog(\frac{\pi}{3}) = \boxed{\frac{\sqrt{3}}{2}}$

$= \sqrt{\dfrac{4-1}{4}} = \dfrac{\sqrt{3}}{2}$

ب)

$f(x)=[x] \to [-3/4] = -4$

$g(x) = \dfrac{x}{1-x} = \dfrac{\sqrt{2}}{1-\sqrt{2}} = \dfrac{\sqrt{2}+2}{1-2} \simeq -3/4$

$fog(\sqrt{2}) = \boxed{-4}$

۳-

$f(x)=\sin x \to \sin(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$

$g(x) = x\sqrt{1-x^2} \to \frac{\sqrt{2}}{2}\sqrt{1-\frac{2}{4}} = \frac{\sqrt{2}}{2}\times\frac{\sqrt{2}}{2} = \frac{2}{4} = \frac{1}{2}$

$gof(\frac{\pi}{4}) = \boxed{\frac{1}{2}}$

۴- الف) $fog(x) = \{(4,5)(2,5)(6,12)(8,12)\}$

ب) $goF(x) = \varnothing$

د) $gog(x) = \{(4,8)(2,6)(6,10)\}$

ج) $FoF(x) = \varnothing$

۵-

$F(x)=\{(2,1)(3,2)(4,5)(1,7)\}$ $\quad g(x)=\{(1,2)(3,1)(a,3)(b,1)\}$

$(4,2)\in fog$

$(4,1)\in gof$

$fog\begin{cases}1\to 2\to 1\\ 3\to 1\to 7\\ a\to 3\to 2\\ b\to 1\to 7\end{cases} \Rightarrow a=4$

$gof\begin{cases}2\to 1\to 2\\ 3\to 2\to \times\\ 4\to 5\to 1\\ 1\to 7\to \times\end{cases} \Rightarrow b=5$

$(a,b) = \boxed{(4,5)}$

$f(f(x)) = 4x+3 \rightarrow a(ax+b)+b = a^2x+ab+b$ $\quad a^2=4 \rightarrow a=2 \text{ یا } a=-2$ $\quad ab+b=3 \rightarrow b(a+1)=3 \rightarrow b = 1, -3$ -۶

$g(2x+3) = 3x-2 \rightarrow a(2x+3)+b = 2ax+3a+b = 3x-2 \longrightarrow a=\frac{3}{2},\ b=\frac{-13}{2}$ تو بالایی حل کردم

$\cancel{gof(-1)}$

$f_{(x)} = 2x+1,\quad g_{(x)} = \frac{3}{2}x - \frac{13}{2}$

$f(-1) = -2+1 = -1,\quad g(-1) = -\frac{3}{2} - \frac{13}{2} = \frac{-16}{2} = -8$

$gof(-1) = \boxed{-8}$

-۷

$f(x) = \sqrt{x+|x|},\quad g(x) = \frac{1}{x^2-4x}$

$D_{gof} = \{x \in D_f,\ f(x) \in D_g\} \rightarrow \begin{cases} x \in \mathbb{R} \\ \sqrt{x+|x|} \in \mathbb{R} - \{0, 4\} \end{cases} \rightarrow D_{gof} = (0, +\infty) - \{8\}$

$D_f = x+|x| \geq 0 \rightarrow \mathbb{R}$

$D_g = x^2-4x \neq 0 \rightarrow x(x-4) \neq 0 \begin{cases} x \neq 4 \\ x \neq 0 \end{cases}$ $\quad x+|x| \neq 0 \rightarrow x \neq (-\infty, 0]$ $\quad x+|x| \neq 16 \rightarrow x \neq 8$

-۸

$f(x) = \sqrt{1-x^2},\quad g(x) = \sqrt{x}$ $\quad 0 \leq 1-x^2 \leq 1 \rightarrow 1 \geq x^2 \geq 0 \rightarrow x \in [-1, 1]$

$D(f+g)of = \{x \in D_f,\ f(x) \in D_{f+g}\} \rightarrow \{x \in [-1,1],\ x \in [-1,1]\}$ $\quad D = [-1, 1]$

$D_{f+g} = D_f \cap D_g = [-1,1] \cap [0,+\infty) = [0, 1]$

$D_g = x \geq 0,\quad D_f = [-1, 1]$

-۹

الف) $f\left(\frac{3x+1}{x-2}\right) = 4x+5$

$\frac{3x+1}{x-2} = t \rightarrow x = \frac{2t+1}{t-3}$

$f(t) = 4\left(\frac{2t+1}{t-3}\right)+5 = \frac{8t+4}{t-3}+5$

$f(x) = \frac{8x+4}{x-3}+5$

ب) $f\left(x+\frac{1}{x}\right) = x^3 + \frac{1}{x^3}$

$x+\frac{1}{x} = t \Rightarrow \left(x+\frac{1}{x}\right)^3 - 3\left(x+\frac{1}{x}\right)$

$f(t) = t^3 - 3t$

$f(x) = x^3 - 3x$

۱۰

$f(x) = x\sqrt{x}$

$gof(x) \rightarrow g(f(x)) = 0$ $\quad f(x) = 1 \rightarrow x\sqrt{x} = 1 \rightarrow x = 1$

$g(1) = 0 \quad g(2\sqrt{2}) = 0$ $\quad f(x) = 2\sqrt{2} \rightarrow x\sqrt{x} = 2\sqrt{2} \rightarrow x = 2$

اختلاف طول نقاطی که نمودار gof محور xها را قطع می‌کند $= x_2 - x_1 = 2-1 = \boxed{1}$