سانیا جاوری

تکلیف سری ۹ سامس B یازدهم ۱۹,۲۵

۱) $\cot \frac{\frac{2}{3}\pi}{\frac{2}{1}} = \cot \frac{1}{3}\pi = \cot 30^\circ \Rightarrow \sqrt{3} \longrightarrow \sqrt{(\sqrt{3})^2+1} = \sqrt{4} = 2$ ۳

۲) الف)

$f(g(\frac{\pi}{3})) \Rightarrow g(\frac{\pi}{3}) = \frac{1}{2}$

$f(\frac{1}{\frac{1}{x}}) = \sqrt{\frac{\frac{2}{x}-1}{\frac{1}{x^2}}} \longrightarrow f(\frac{1}{2}) = \sqrt{\frac{\frac{2}{\frac{1}{2}}-1}{\frac{1}{(\frac{1}{2})^2}}} = \sqrt{\frac{4-1}{4}} = \frac{\sqrt{3}}{2}$ ۵

ب)

$f(g(\sqrt{2})) \Rightarrow g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}} \times \frac{(1+\sqrt{2})}{(1+\sqrt{2})} = -2-\sqrt{2}$

$f(-2-\sqrt{2} = [-2-\sqrt{2}] \longrightarrow f(-2-\sqrt{2}) = [-4]$

۳) $g(f(\frac{\pi}{4})) \Rightarrow f(\frac{\pi}{4}) = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2} \to g(\frac{\sqrt{2}}{2}) = \frac{\sqrt{2}}{2} \times \sqrt{1-(\frac{\sqrt{2}}{2})^2} = \sqrt{\frac{2}{4}} = \frac{\sqrt{2}}{2}$ ۵

$\frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} = \frac{2}{4} = \frac{1}{2}$

۴) الف) $f(g(2,4,6,8))$

$f(g(4)) = f(6) = 5$ $\quad f(g(6)) = f(8) = 12$

$f(g(2)) = f(4) = 5$ $\quad f(g(10)) = f(10) = 12$

ب) $g(f(4,6,8,10)) = \cancel{\emptyset}$

ج) $f(f(x)) = \cancel{\emptyset} \longrightarrow f(12) = \cancel{\emptyset}$

$4,6,8,10$ $\quad f(5) = \cancel{\emptyset}$ ۵

د) $g(g(6,4,8,2))$

$g(g(6)) = 8 \longrightarrow g(8) = 10$

$g(g(8)) = 10 \longrightarrow g(10) = \cancel{\emptyset}$

$g(g(4)) = 6 \longrightarrow g(6) = 8$

$g(g(2)) = 4 \longrightarrow g(4) = 6$

(5) 1,75

$(4,2)\in f\circ g \rightarrow f\circ g(4)=2$

$f\circ g(x)=f(g(x))$

$f(g(4))=2$

$f(3)=2$

$g(4)=3 \longrightarrow \underline{a=4} \Rightarrow (a,3)=(4,3)$

$(4,1)\in g\circ f$

$g\circ f(4)=1$

$g(f(4))=1 \longrightarrow f(4)=5,\ g(5)=1 \quad (3,1)=(b,1) \longrightarrow \underline{b=5}$

$(a,b)=(4,5)$

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(6) 2

$f(f(x))=4x+3$

$g(2x+3)=3x-2 \longrightarrow f(x)=ax+b$

$f(f(x))=4x+3$

$f(f(x))=a(ax+b)+b=a^2x+ab+b$

$4x+3=$

$a^2=4$

$ab+b=3 \longrightarrow a=2,\ a=-2$

$a=2 \rightarrow 2b+b=3 \Rightarrow 3b=3 \rightarrow b=1 \rightarrow f(x)=2x+1$

$a=-2 \rightarrow (-2)b+b=3 \Rightarrow -b=3 \rightarrow b=-3 \rightarrow f(x)=-2x-3$

$g(2x+3)=3x-2 \rightarrow t=2x+3 \rightarrow x=\frac{t-3}{2} \quad g(t)=3\left(\frac{t-3}{2}\right)-2=\frac{3t-9}{2}-2=\frac{3t-13}{2}=g(t)$

$g(f(x)) \quad f(x)=2x+1 \rightarrow g(f(x))=g(2x+1)=\frac{3(2x+1)-13}{2}=\frac{6x+3-13}{2}=\frac{6x-10}{2}=3x-5 \quad (-1)=-8$

$f(x)=-2x-3 \rightarrow g(f(x))=g(-2x-3)=\frac{3(-2x-3)-13}{2}=\frac{-6x-9-13}{2}=\frac{-6x-22}{2}=-3x-11 \quad (-1)=-8$

$\boxed{g(f(-1))=-8}$

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(7) 2

$f(x)=\sqrt{x+|x|} \Rightarrow |x|=\begin{cases} x & x\geq 0 \\ -x & x<0 \end{cases}$

$f(x)=\begin{cases} \sqrt{x+x}=\sqrt{2x} \rightarrow 2x\geq 0 \rightarrow x\geq 0 \\ \sqrt{x-x}=\sqrt{0}=0 \quad x<0 \end{cases}$

$D_f=\mathbb{R}$

$g(x)=\frac{1}{x^2-4x} \rightarrow x(x-4) \qquad f(x)\in D_g \rightarrow f(x)\neq 0,4$

$f(x)=0 \rightarrow x<0 \qquad D_g=\mathbb{R}-\{0,4\}$

$f(0)=\sqrt{0}=0 \rightarrow x=0 \qquad f(x)=4 \rightarrow x\geq 0 \rightarrow \sqrt{2x}=4 \Rightarrow 2x=16 \Rightarrow x=8$

$D_{g\circ f}=(0,+\infty)-\{8\}$

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(8) 1,5

$(f+g)\circ f=\sqrt{1-(f(x))^2}+\sqrt{f(x)} \rightarrow f(x)=\sqrt{1-x^2} \rightarrow (\sqrt{1-x^2})^2=1-x^2$

(1) $1-x^2\geq 0 \rightarrow -1\leq x\leq 1 \cap D_g=x\geq 0 \rightarrow 0\leq x\leq 1 \qquad 0\leq\sqrt{1-x^2}\leq 1 \rightarrow x^2\geq 0$ ✓

(2) $g(f(x))=\sqrt{f(x)} \longrightarrow f(x)\geq 0 \longrightarrow f(x)=\sqrt{1-x^2}\geq 0$

$\boxed{D=[-1,1]}$

$D_{(f+g)\circ f}=[-1,1]$

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(9)

الف) $\frac{3x+1}{x-2}=t \rightarrow tx-2t=3x+1 \rightarrow tx-3x=2t+1$

$x(t-3)=2t+1$

$x=\frac{t-3}{2t+1}$

$\Rightarrow f(t)=\frac{8t+4}{t-3}+\frac{5t-15}{t-3}=\frac{13t-11}{t-3}$

ب) $x+\frac{1}{x}=t \rightarrow \left(x+\frac{1}{x}\right)^3=x^3+3x\frac{1}{x}-3x\frac{1}{x^2}+\frac{1}{x^3}$

$x^3+\frac{1}{x^3}=\left(x+\frac{1}{x}\right)^3-3\left(x+\frac{1}{x}\right) \longrightarrow x+\frac{1}{x}=t \rightarrow f(t)=t^3-3t$

$f(x)=x^3-3x$

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$= x^3 - 3x$

$g(1) = 0$

$g(2\sqrt{2}) = 0$

$f(x) = x\sqrt{x}$

$g \circ f(x) \Rightarrow g(f(x)) = 0$

$x\sqrt{x} = 1$

الف) $x^{\frac{3}{2}} = 1 \longrightarrow x = 1$

ب) $x^{\frac{3}{2}} = 2\sqrt{2} = 2 \times 2^{\frac{1}{2}} = 2^{\frac{3}{2}} \Rightarrow x^{\frac{3}{2}} = 2^{\frac{3}{2}} \Rightarrow x = 2$

$x_2 = 2 \qquad x_1 = 1$

$|x_2 - x_1| = 2 - 1 = 1$

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