۱)

$$f(x)=\begin{cases}\cot\frac{\pi x}{4} & ; x\leq 1\\ \sqrt{x^2+1} & ; x>1\end{cases}$$

$$\frac{\cot \pi \times \frac{2}{3}}{4} \Rightarrow \frac{1}{6}\cot\pi$$

(۲)

$$fof\left(\frac{2}{3}\right) \Rightarrow \cot\frac{\pi}{6} = \sqrt{3} \qquad f(\sqrt{3})=\sqrt{3+1}=\sqrt{4}=2$$

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۲) الف)

$$f\left(\frac{1}{x}\right)=\sqrt{\frac{2x-1}{x^2}} \Rightarrow f\left(g\left(\frac{\pi}{3}\right)\right)=f\left(2\cos^2(60^\circ)\right)=f\left(\frac{1}{2}\right)=\frac{\sqrt{2}}{2}$$

$$g(x)=2\cos^2 x$$

(۲)

ب)

$$f(x)=[x] \qquad g(x)=\frac{x}{1-x} \Rightarrow fog(\sqrt{2}) \Rightarrow f(g(\sqrt{2})) \Rightarrow g(\sqrt{2})=\frac{\sqrt{2}}{1-\sqrt{2}}\times\frac{1+\sqrt{2}}{1+\sqrt{2}}$$

$$=\frac{\sqrt{2}+2}{1-2}=-\sqrt{2}-2 \qquad f(-\sqrt{2}-2)=[-\sqrt{2}-2]=[-3/4]=-4$$

۳)

$$f(x)=\sin x \qquad g(x)=x\sqrt{1-x^2}$$

$$gof\left(\frac{\pi}{4}\right)=g\left(f\left(\frac{\pi}{4}\right)\right) \Rightarrow f\left(\frac{\pi}{4}\right)=\sin\frac{\pi}{4} \Rightarrow \frac{\sqrt{2}}{2}$$

(۲)

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$$g\left(\frac{\sqrt{2}}{2}\right)=\frac{\sqrt{2}}{2}\sqrt{1-\frac{1}{2}} \Rightarrow \frac{\sqrt{2}}{2}\times\frac{\sqrt{1}}{\sqrt{2}}=\boxed{\frac{1}{2}}$$

۴)

$$f(x)=\{(4,5),(6,5),(8,12),(10,12)\} \qquad g(x)=\{(4,6),(2,4),(6,8),(8,10)\}$$

الف) $fog(x)=f(g(x))$ $\quad 4\to 6\to 5 \;/\; 2\to 4\to 5 \;/\; 6\to 8\to 12$

$8\to 10\to 12 \to fog(x)=\{(4,5),(2,5),(6,12),(8,12)\}$

(۲)

ب) $gof(x) \Rightarrow g(f(x)) = 4\to 5 \;/\; 6\to 5 \;/\; 8\to 12 \;/\; 10\to 12 \quad \emptyset$

ج) $fof(x)=f(f(x)) = 4\to 5 \;/\; 6\to 5 \;/\; 8\to 12 \;/\; 10\to 12 \quad \emptyset$

د) $gog(x)=\{(4,8),(2,6),(6,10)\}$

$f(g(4))=2 \Rightarrow a=4$ $\quad (4,5)$ $\quad$ 3 $\quad$ 5

$g(\cancel{f}(4))=1 \Rightarrow g(5)=1 \quad b=5$

$f(f(x))=4x+3 \qquad gof(-1)=g(f(-1))=?$ $\quad$ 1 $\quad$ 6

$f(f(x))=f(ax+b)=a(ax+b)+b=a^2x+ab+b$

$f(f(x))=4x+3 \Rightarrow a^2x+ab+b=4x+3 \qquad a^2=4 \qquad \boxed{a=\pm 2}$

$ab+b=3 \Rightarrow b(a+1)=3$

$\to b=+1 \quad f(x)=2x+1 \xrightarrow{x=-1} -1$

$\to b=-3 \quad f(x)=-2x-3 \xrightarrow{x=-1} -1$

$g(f(-1))=? \qquad \overbrace{3(2x+1)-2}^{6x+1} \qquad g(2x+1)$

$\to f(-1)=6(-1)+1=-5$

$\to f(-1)=-6(-1)-11=-5$ $\quad$ جواب:

$g(-2x-3) \Rightarrow 3(-2x-3)-2$

$-6x-1$

$g(-1)=-8$

$f(x)=\sqrt{x+|x|} \qquad g(x)=\frac{1}{x^2-4x} \qquad gof(x)=$ $\quad$ 7

$Dgof=\{x\in D_f, f(x)\in D_g\} \quad D_f=x+|x|\geq 0 \quad D_f=(-\infty,+\infty)$

3

$g(x)=\frac{1}{x^2-4x} \Rightarrow x^2-4x\neq 0 \qquad D_g=\mathbb{R}-\{0,4\}$

$x(x-4)\neq 0$

$0 \quad 4$

$f(x)\neq 0,4 \Rightarrow \begin{cases} f(x)=0 / \sqrt{x+|x|}=0 \\ f(x)=4 \quad \sqrt{x+|x|} \end{cases}$

$Dgof=\{x\in \mathbb{R}, x>0, x\neq 8\}$

$x\geq 0 \begin{cases} 2x=16 \quad x=8 \\ x+(-x)=0 \ldots 16 \end{cases}$

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$Dgof=(0,+\infty)-\{+8\}$

۱۸) $f(x)=\sqrt{1-x^2}$ $\quad g(x)=\sqrt{x}$ $\quad (f+g)\circ f$

$1-x^2\geq 0 \quad x^2\leq 1 \Rightarrow -1\leq x\leq 1 \quad D_f=[-1,1]$

$D_g=[0,+\infty) \quad D_f\cap D_g=[-1,1]\cap[0,+\infty)=[0,1]$ ۳

$D_{(f+g)\circ f}=\{x\in D_f \mid f(x)\in D_{f+g}\}$

$0\leq\sqrt{1-x^2}\leq 1 \Rightarrow 0\leq 1-x^2\leq 1 \Rightarrow 1-x^2\geq 0 \quad x^2\leq 1$

$-1\leq x\leq 1$

$1-x^2\leq 1 \Rightarrow -x^2\leq 0 \Rightarrow x^2\geq 0$ همواره برقرار

$\boxed{D_{(f+g)\circ f}=[-1,1]\cap[-1,1]=[-1,1]} \leftarrow$ جواب

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۱۹) الف) $f\left(\frac{3x+1}{x-2}\right)=4x+5 \quad \frac{3x+1}{x-2}=t \quad 3x+1=tx-2t$

$3x-tx=-2t-1 \Rightarrow x(3-t)=-(2t+1) \quad f(t)=4\left(\frac{2t+1}{t-3}\right)+5 \Rightarrow$

$\frac{8t+4}{t-3}+\frac{5(t-3)}{t-3} \Rightarrow f(t)=\frac{13t-11}{t-3} \rightarrow f(x)=\frac{13x-11}{x-3}$

ب) $f\left(x+\frac{1}{x}\right)=x^3+\frac{1}{x^3} \quad x+\frac{1}{x}=t$ ۳

$\left(x+\frac{1}{x}\right)^3=x^3+3x^2\frac{1}{x}+3x\frac{1}{x^2}+\frac{1}{x^3}$

$\Rightarrow x^3+3x+\frac{3}{x}+\frac{1}{x^3} \Rightarrow x^3+\frac{1}{x^3}=\underbrace{\left(x+\frac{1}{x}\right)}_{t}^3-3\underbrace{\left(x+\frac{1}{x}\right)}_{t}$

$\Rightarrow t^3-3t \Rightarrow f(x)=x^3-3x$

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۲۰) $g(1)=0 \quad f(x)=1 \quad x\sqrt{x}=1 \Rightarrow x=1$ $\quad |2-1|=1$ ۲

$g(2\sqrt{2})=0 \quad f(x)=2\sqrt{2} \quad x\sqrt{x} \Rightarrow x=2$