۱)

$$f(x)=\begin{cases}\cot\frac{\pi x}{4} & ; x\leq 1\\ \sqrt{x^2+1} & ; x>1\end{cases} \qquad \frac{\cot \pi \times \frac{2}{3}}{4} \Rightarrow \frac{1}{6}\cot\pi$$

$$f\circ f\left(\frac{2}{3}\right) \Rightarrow \cot\frac{\pi}{6} = \sqrt{3} \qquad f(\sqrt{3})=\sqrt{3+1}=\sqrt{4}=2$$

(زاویه °30)

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۲) الف)

$$f\left(\frac{1}{x}\right)=\sqrt{\frac{2x-1}{x^2}} \Rightarrow f\left(g\left(\frac{\pi}{3}\right)\right)=f\left(2\cos^2(60^\circ)\right)=f\left(\frac{1}{2}\right)=\frac{\sqrt{2}}{2}$$

$$g(x)=2\cos^2 x$$

ب)

$$f(x)=[x] \qquad g(x)=\frac{x}{1-x}$$

$$\Rightarrow f\circ g(\sqrt{2}) \Rightarrow f(g(\sqrt{2})) \Rightarrow g(\sqrt{2})=\frac{\sqrt{2}}{1-\sqrt{2}}\times\frac{1+\sqrt{2}}{1+\sqrt{2}}$$

$$=\frac{\sqrt{2}+2}{1-2}=-\sqrt{2}-2 \qquad f(-\sqrt{2}-2)=[-\sqrt{2}-2]=[-3/4]=-4$$

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۳)

$$f(x)=\sin x \qquad g(x)=x\sqrt{1-x^2}$$

$$g\circ f\left(\frac{\pi}{4}\right)=g\left(f\left(\frac{\pi}{4}\right)\right)\Rightarrow f\left(\frac{\pi}{4}\right)=\sin\frac{\pi}{4}\Rightarrow\frac{\sqrt{2}}{2}$$

$$g\left(\frac{\sqrt{2}}{2}\right)=\frac{\sqrt{2}}{2}\sqrt{1-\frac{1}{2}}\Rightarrow\frac{\sqrt{2}}{2}\times\frac{\sqrt{2}}{\sqrt{2}\cdot 2}=\frac{1}{2}$$

(°45)

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۴)

$$f(x)=\{(4,5),(6,5),(8,12),(10,12)\} \qquad g(x)=\{(4,6),(2,4),(6,8),(8,10)\}$$

الف) $f\circ g(x)=f(g(x))$: $4\to 6\to 5$ / $2\to 4\to 5$ / $6\to 8\to 12$

$8\to 10\to 12 \;\to\; f\circ g(x)=\{(4,5),(2,5),(6,12),(8,12)\}$

ب) $g\circ f(x)\Rightarrow g(f(x))$ = $4\to 5$ / $6\to 5$ / $8\to 12$ / $10\to 12$ $\quad \varnothing$

ج) $f\circ f(x)=f(f(x))$ = $4\to 5$ / $6\to 5$ / $8\to 12$ / $10\to 12$ $\quad \varnothing$

د) $g\circ g(x)=\{(4,8),(2,6),(6,10)\}$

$f(g(\varepsilon)) = 2 \Rightarrow a = \varepsilon$ $\quad (\varepsilon, d)$

$g(f(\cancel{2})) = 1 \Rightarrow g(0) = 1 \quad b = d$

$f(f(x)) = 4x + 3 \qquad gof(-1) = g(f(-1)) = ?$

$f(f(x)) = f(ax+b) = a(ax+b) + b = a^2x + ab + b$

$f(f(x)) = 4x + 3 \Rightarrow a^2x + ab + b = 4x + 3 \qquad a^2 = 4 \qquad a = \pm 2$

$ab + b = 3 \Rightarrow b(a+1) = 3$

- $b = +1 \quad f(x) = 2x + 1$
- $b = -3 \quad f(x) = -2x - 3$

$g(f(-1)) = ?$ $\qquad \overbrace{3(2x+1) - 2}^{6x+1}$ $\qquad g(2x+1)$

$g(-2x-3) \Rightarrow 3(-2x-3) - 2$

$-6x - 11$

- $f(-1) = 6(-1) + 1 = (-5)$
- $f(-1) = -6(-1) - 11 = (-5)$ ؟!

$f(x) = \sqrt{x + |x|} \qquad g(x) = \dfrac{1}{x^2 - 4x} \qquad gof(x) = ?$

$D_{gof} = \{x \in D_f \,,\, f(x) \in D_g\} \qquad D_f = x + |x| \geq 0 \qquad D_f = (-\infty, +\infty)$

$g(x) = \dfrac{1}{x^2 - 4x} \Rightarrow x^2 - 4x \neq 0 \qquad D_g = \mathbb{R} - \{0, 4\}$

$x(x-4) \neq 0$

$x \neq 0, 4$

$\rightarrow f(x) \neq 0, 4 \Rightarrow \begin{cases} f(x) = 0 \,/\, \sqrt{x + |x|} = 0 \\ f(x) = 4 \,/\, \sqrt{x + |x|} \end{cases}$

$D_{gof} = \{x \in \mathbb{R}, x > 0 \,,\, x \neq 8\}$

$D_{gof} = (0, +\infty) - \{8\}$

$x \geq 0 \begin{cases} 2x = 16 \quad x = 8 \\ x + (-x) = 0 \neq 16 \end{cases}$

ق ق ق

۸

$f(x)=\sqrt{1-x^2}$ $g(x)=\sqrt{x}$ $(f+g)\circ f$

$1-x^2\geq 0 \quad x^2\leq 1 \Rightarrow -1\leq x\leq 1 \quad D_f=[-1,1]$

$D_g=[0,+\infty) \quad D_f\cap D_g=[-1,1]\cap[0,+\infty)=[0,1]$

$D_{(f+g)\circ f}=\{x\in D_f \mid f(x)\in D_{f+g}\}$

$0\leq\sqrt{1-x^2}\leq 1 \Rightarrow 0\leq 1-x^2\leq 1 \Rightarrow 1-x^2\geq 0 \quad x^2\leq 1$

$-1\leq x\leq 1$

$1-x^2\leq 1 \Rightarrow -x^2\leq 0 \Rightarrow x^2\geq 0$ همواره برقرار

$D_{(f+g)\circ f}=[-1,1]\cap[-1,1]=[-1,1]$ ← جواب

۹

الف) $f\left(\frac{2x+1}{x-2}\right)=4x+5 \quad \frac{2x+1}{x-2}=t \quad 2x+1=tx-2t$

$2x-tx=-2t-1 \Rightarrow x(2-t)=-(2t+1) \quad f(t)=4\left(\frac{2t+1}{t-2}\right)+5 \Rightarrow$

$\frac{8t+4}{t-2}+\frac{5(t-2)}{t-2} \Rightarrow f(t)=\frac{13t-11}{t-2} \rightarrow f(x)=\frac{13x-11}{x-2}$

ب) $f\left(x+\frac{1}{x}\right)=x^3+\frac{1}{x^3} \quad x+\frac{1}{x}=t$

$\left(x+\frac{1}{x}\right)^3=x^3+3x^2\frac{1}{x}+3x\frac{1}{x^2}+\frac{1}{x^3}$

$\Rightarrow x^3+3x+\frac{3}{x}+\frac{1}{x^3} \Rightarrow x^3+\frac{1}{x^3}=\underbrace{\left(x+\frac{1}{x}\right)^3}_{t^3}-3\underbrace{\left(x+\frac{1}{x}\right)}_{t}$

$\Rightarrow t^3-3t \Rightarrow f(x)=x^3-3x$

۱۰

$g(1)=0 \quad f(x)=1 \quad x\sqrt{x}=1 \Rightarrow x=1$

$g(3\sqrt{3})=0 \quad f(x)=3\sqrt{3} \quad x\sqrt{x} \Rightarrow x=3$

$|3-1|=2$