نام و نام خانوادگی ...الینا علی‌نجاری... پاسخنامه تشریحی تکلیف شماره ...۹... کلاس ...یازدهم ریاضی B...

$f(x)=\begin{cases}\cot\frac{\pi x}{4} ; & x\leq 1\\ \sqrt{x^2+1} ; & x>1\end{cases}$ ۱

$f\circ f(\frac{2}{3})$

$f(\frac{2}{3})=\cot\frac{2\pi}{12}=\cot\frac{\pi}{6}=\sqrt{3}$

$f(\sqrt{3}) \xrightarrow{\sqrt{3}>1} \sqrt{(\sqrt{3})^2+1}=2$

الف) ۲

$f\circ g(\frac{\pi}{3}) \to g(\frac{\pi}{3})=2\underbrace{\cos^2(\frac{\pi}{3})}_{(\frac{1}{2})^2}=\frac{1}{2}$

$f(\frac{1}{2}) \to f(\frac{1}{x})=\sqrt{\frac{2x-1}{x^2}} \xrightarrow{x=2} f(\frac{1}{2})=\sqrt{\frac{4-1}{4}}$

$f\circ g(\frac{\pi}{3})=\frac{\sqrt{3}}{2}$

ب)

$f\circ g(\sqrt{2}) \to g(\sqrt{2})=\frac{\sqrt{2}}{1-\sqrt{2}} \to \frac{\sqrt{2}}{1-\sqrt{2}}\times\frac{1+\sqrt{2}}{1+\sqrt{2}}$

$=\frac{\sqrt{2}+2}{1-2}=-\sqrt{2}-2 \cong -3/4$

$f(-\sqrt{2}-2)=[-\underbrace{\sqrt{2}}_{\cong 1/4}-2]=-4$

$f(x)=\sin x \quad g(x)=x\sqrt{1-x^2} \quad g\circ f(\frac{\pi}{4})$ ۳

$f(\frac{\pi}{4})=\sin\frac{\pi}{4}=\frac{\sqrt{2}}{2}$

$g(\frac{\sqrt{2}}{2})=\frac{\sqrt{2}}{2}\sqrt{\underbrace{1-(\frac{\sqrt{2}}{2})^2}_{1-\frac{1}{2}}}=\frac{\sqrt{2}}{2}\sqrt{\frac{1}{2}}=\frac{\sqrt{2}}{2}\times\frac{\sqrt{2}}{2}=\frac{1}{2}$

الف) $f\circ g(x)=\{(4,5),(2,5),(6,12),(8,12)\}$ ۴

ج) $f\circ f(x)=\varnothing$

ب) $g\circ f(x)=\varnothing$

د) $g\circ g(x)=\{(4,8),(2,6),(6,10)\}$

$f=\{(2,1),(3,2),(4,5),(1,7)\} \quad g=\{(1,2),(3,1),(a,3),(b,1)\}$ ۵

$(4,2)\in f\circ g \quad (4,1)\in g\circ f \quad (a,b)? \longrightarrow (a,b)=(4,5)$

درون (g) بیرونی f

بیرونی f

$f(g(4))=2$

$g(\underbrace{f(4)}_{5})=1$

$g(4)=3 \to a=4$

$g(5)=1 \to b=5$

۶

f و g توابعی اند: $f(f(x)) = 4x+3$, $g(2x+3) = 3x-2$, $g\circ f(-1) = ?$

$g(2x+3) = 3x-2$
$2x+3 = t$
$x = \frac{t-3}{2}$
$g(x) = \frac{3x-13}{2}$

$f(x) = ax+b$
$4x+3 = a(ax+b)+b$
$4x+3 = a^2x + ab + b$
$4 = a^2 \rightarrow a = \pm 2$
$a=-2: \; ab+b=3 \rightarrow -b=3 \rightarrow b=-3 \rightarrow f(x) = -2x-3 \xrightarrow{x=-1} = -1$
$a=2: \; ab+b=3 \rightarrow 3b=3 \rightarrow b=1 \rightarrow f(x) = 2x+1 \xrightarrow{x=-1} = -1$

$g(-1) = -8$

۷

$g(x) = \frac{1}{x^2-4x}$, $f(x) = \sqrt{x+|x|}$, $D_{g\circ f} = ?$

$\frac{1}{x(x-4)} \rightarrow D_g = \mathbb{R} - \{0,4\}$

(1) $D_f = \mathbb{R}$

$\{x \in D_f : f(x) \in D_g\}$

(2) $\sqrt{x+|x|} \neq \{0,4\}$
$\sqrt{x+|x|} \neq 0 \rightarrow x > 0$
$\sqrt{x+|x|} \neq 4 \rightarrow x+|x| \neq 16 \xrightarrow{x>0} 2x \neq 16 \rightarrow x \neq 8$

$D_{g\circ f} = (0,+\infty) - \{8\}$

۸

$f(x) = \sqrt{1-x^2}$, $g(x) = \sqrt{x}$, $D_{(f+g)\circ f} ? = [-1,1]$

$D_f \Rightarrow (1-x)(1+x)$
$D_f = [-1,1]$ (1)
$D_g = [0,+\infty)$

$\{x \in D_f : f(x) \in D_{f+g}\}$

$\sqrt{1-x^2} \in [0,1] \rightarrow 0 \leq \sqrt{1-x^2} \leq 1 \xrightarrow{\text{توان ۲}} 0 \leq 1-x^2 \leq 1$
$\rightarrow -1 \leq -x^2 \leq 0 \rightarrow 1 \geq x^2 \geq 0 \rightarrow -1 \leq x \leq 1$ (2)

$D_{(f+g)} \Rightarrow D_f \cap D_g \rightarrow [-1,1] \cap [0,\infty) \rightarrow [0,1]$

۹

الف) $f\left(\frac{3x+1}{x-2}\right) = 4x+5$

$\frac{3x+1}{x-2} = t \rightarrow x = \frac{-2t-1}{t-3} = \frac{2t+1}{t-3}$

$f(x) = 4\left(\frac{2x+1}{x-3}\right)+5 = \frac{8x+4}{x-3}+5$

$= \frac{8x+4+5x-15}{x-3} = \frac{13x-11}{x-3}$

ب) $f\left(x+\frac{1}{x}\right) = x^3+\frac{1}{x^3}$

$x+\frac{1}{x} = t$

$x^3+\frac{1}{x^3}+3\left(x+\frac{1}{x}\right) = t^3$

$x^3+\frac{1}{x^3} = t^3-3t \rightarrow f(x) = x^3-3x$

۱۰

$g(1) = 0 \rightarrow g(f(x)) = 0 \rightarrow f(x) = x\sqrt{x} = 1 \rightarrow x_1 = 1$

$g(2\sqrt{2}) = 0 \rightarrow g(f(x)) = 0 \rightarrow f(x) = x\sqrt{x} = 2\sqrt{2} \rightarrow x_2 = 2$

$x_2 - x_1 = 2 - 1 = 1$