

$$f(x) = \begin{cases} \cot \frac{\pi x}{4} & ; x \leq 1 \\ \sqrt{x^2 + 1} & ; x > 1 \end{cases} \quad f \circ f\left(\frac{1}{4}\right) = ? \quad (2)$$

$$F\left(f\left(\frac{1}{4}\right)\right) = ? \Rightarrow f(\sqrt{3})$$

$$\frac{1}{4} \leq 1 \Rightarrow \cot\left(\frac{\pi \times \frac{1}{4}}{4}\right) = \cot\left(\frac{\pi}{16}\right) = \sqrt{3}$$

$$\sqrt{3} > 1 \Rightarrow \sqrt{(\sqrt{3})^2 + 1} = \sqrt{4} = 2$$

$$\text{الف) } f \circ g\left(\frac{\pi}{4}\right) = f\left(\frac{1}{4}\right) = \sqrt{\frac{1}{16} - 1} \Rightarrow x = 2 \Rightarrow \sqrt{\frac{4-1}{4}} = \frac{\sqrt{3}}{2}$$

$$\cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} \quad \frac{1}{x} = \frac{1}{\sqrt{2}}$$

$$g(x) = x \times \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}$$

$$\Rightarrow f \circ g\left(\frac{\pi}{4}\right) = -\frac{1}{2}$$

$$\left[\frac{x}{1-x}\right] \Rightarrow \left[\frac{\sqrt{2}}{1-\sqrt{2}}\right] \Rightarrow [-2-\sqrt{2}] = -\frac{1}{2} \quad \frac{\sqrt{2}}{1-\sqrt{2}} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} = \frac{2+\sqrt{2}}{1-2} = -2-\sqrt{2}$$

$$f(x) = \sin x \quad g(x) = x \sqrt{1-x^2} \quad g \circ f\left(\frac{\pi}{4}\right) = ? \quad \frac{1}{\sqrt{2}}$$

$$g(f(x)) = \sin x \sqrt{\frac{1-\sin^2 x}{\cos^2 x}} = \sin x \cos x \Rightarrow x = \frac{\pi}{4}$$

$$\frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} = \left(\frac{1}{2}\right)$$

$$\text{الف) } f \circ g(x) = \{(1, 2), (2, 1), (3, 4), (4, 3)\}$$

$$\Rightarrow g \circ f(x) = \{ \} \subseteq \emptyset$$

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$$f = \{(1, 1), (2, 2), (3, 3), (4, 4)\} \quad g = \{(1, 2), (2, 1), (3, 4), (4, 3)\}$$

$$(1, 2) \in f \circ g \quad , \quad (1, 1) \notin g \circ f \quad (a, b) \Rightarrow (1, 3)$$

$$f \rightarrow 1 \rightarrow 2 \quad f \rightarrow 2 \rightarrow 1$$

$$a \rightarrow 3 \rightarrow 4 \quad \omega = b$$

$$f(f(x)) = 5x + 3 \Rightarrow f(ax+b) \Rightarrow a(ax+b) + b \Rightarrow a^2x + ab + b = 5x + 3$$

$$g(2x+3) = 3x-1 \Rightarrow a=2, b=3 \Rightarrow 2x+1$$

$$a(2x+3) + b = 3x-1 \Rightarrow 2ax + 3a + b = 3x-1 \Rightarrow a = \frac{3}{2}, b = -\frac{11}{2}$$

$$g(f(-1)) = -1$$

$$f(x) = \sqrt{x+1} \quad x \geq 0$$

$$g(x) = \frac{1}{x^2 - 5x}$$

$$D_{g \circ f} = \{ (0, +\infty) - \{1\} \}$$

$$1) f(x) = \sqrt{2x}$$

$$2) f(x) = \sqrt{0} = 0$$

$$f(x) = \begin{cases} \sqrt{2x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

$$g(f(x)) = \frac{1}{(f(x))^2 - 5f(x)}$$

$$f(x) \neq 0, f(x) \neq 5$$

$$\begin{aligned} & f(x) \neq 0 \Rightarrow \sqrt{2x} \neq 0 \Rightarrow x \neq 0 \\ & f(x) \neq 5 \Rightarrow \sqrt{2x} \neq 5 \Rightarrow 2x \neq 25 \Rightarrow x \neq \frac{25}{2} \\ & x < 0 \Rightarrow \text{ممنوع} \\ & f(x) = 0 \Rightarrow x \end{aligned}$$

$$F(x) = \sqrt{1-x^2}$$

$$g(x) = \sqrt{x}$$

$$D_{(F+g) \circ f} = \{ \}$$

$$F \circ f + g \circ f \Rightarrow F(f(x)) + g(f(x))$$

$$\Rightarrow \sqrt{1-x^2} \Rightarrow F(x) = \sqrt{1-x^2}$$

$$D_F = \{1-x^2 \geq 0\}$$

$$\sqrt{1-(\sqrt{1-x^2})^2} \Rightarrow \sqrt{1-(1-x^2)} = \sqrt{x^2} = |x|$$

$$-1 \leq x \leq 1$$

$$D = [-1, 1]$$

$$الف) f\left(\frac{3x+1}{x-2}\right) = 5x+10$$

$$\frac{3x+1}{x-2} = t \Rightarrow 3x+1 = tx-2t$$

$$x(3-t) = -2t-1 \Rightarrow x = \frac{-2t-1}{3-t} = \frac{2t+1}{t-3}$$

$$f\left(\frac{2t+1}{t-3}\right) + 10 = 5x+10 \Rightarrow \frac{2t+1}{t-3} + 10 = 5 \cdot \frac{2t+1}{t-3} + 10$$

$$\frac{13t-11}{t-3} = \frac{13x-11}{x-3} = f(x)$$

$$ب) f\left(x + \frac{1}{x}\right) = x^3 + \frac{1}{x^3}$$

$$x + \frac{1}{x} = t$$

$$\left(x + \frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3x \cdot \frac{1}{x} \left(x + \frac{1}{x}\right) = x^3 + \frac{1}{x^3} + 3 \left(x + \frac{1}{x}\right)$$

$$x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)^3 - 3 \left(x + \frac{1}{x}\right)$$

$$x^3 + \frac{1}{x^3} = t^3 - 3t = F(x)$$

$$g(1) = 0, g(\sqrt{2}) = 0$$

$$(g \circ f)(x) \Rightarrow g(f(x)) = 0$$

$$f(x) = 1 \Rightarrow f(x) = \sqrt{2}$$

$$f(x) = x\sqrt{x} = x^1 \times x^{\frac{1}{2}} \Rightarrow x^{\frac{3}{2}}$$

$$x^{\frac{3}{2}} = 1 \Rightarrow x = 1, x^{\frac{3}{2}} = \sqrt{2} \Rightarrow x^{\frac{3}{2}} = \sqrt{2} \Rightarrow x = \sqrt{2}$$

اختلاف نقاط و (gof)(x) و (fgo)(x) و (fgo)(x) و (gof)(x) و (fgo)(x) و (gof)(x)

$$x-1 = 1$$

و (fgo)(x) و (gof)(x) و (fgo)(x) و (gof)(x) و (fgo)(x) و (gof)(x)