

۱) $x^2 - 5x - 1^4 = 0$ $x^2 - 5x - 1^4 = 0$ $(x-9)(x+1) = 0$ $\rightarrow x = \frac{9}{1} = 9$ $\rightarrow x = -\frac{1}{1} = -1$ $x = 9, x = -1$	۲) $x^2 - 4x - 1^4 = 0$ $x^2 - 4x - 1^4 = 0$ $(x-4)(x+1) = 0$ $\rightarrow x = \frac{4}{1} = 4$ $\rightarrow x = -\frac{1}{1} = -1$ $x = 4, x = -1$	۳) $x^2 - 8x - 1^4 = 0$ $x^2 - 8x - 9 = 0$ $(x-9)(x+1) = 0$ $\rightarrow x = \frac{9}{1} = 9$ $\rightarrow x = -\frac{1}{1} = -1$ $x = 9, x = -1$	۴) $x^2 - 5x - 4 = 0$ $x^2 - 5x - 1^4 = 0$ $(x-9)(x+4) = 0$ $\rightarrow x = \frac{9}{1} = 9$ $\rightarrow x = -\frac{4}{1} = -4$ $x = \frac{9}{1}, x = -\frac{4}{1}$	۱
۵) $x^2 + 1^4x - 1^4 = 0$ $\Delta = b^2 - 4ac = 1^4 - 4(1)(-1) = 1 + 4 = 5$ $\Delta > 0 \rightarrow$ ریشه دارد! $x = \frac{-1 \pm \sqrt{5}}{2}$	۶) $x^2 + 4x + 1^4 = 0$ $\Delta = b^2 - 4ac = 4^2 - 4(1)(1) = 16 - 4 = 12$ $\Delta > 0 \rightarrow$ ریشه دارد! $x = \frac{-4 \pm \sqrt{12}}{2} = -2 \pm \sqrt{3}$	۷) $x^2 + 5x + 1^4 = 0$ $x^2 + 5x + 4 = 0$ $(x+2)(x+1) = 0$ $\rightarrow x = -\frac{2}{1} = -2$ $\rightarrow x = -\frac{1}{1} = -1$ $x = -2, x = -1$	۸) $x^2 - 5x - 1^4 = 0$ $(x-7)(x+2) = 0$ $\rightarrow x = 7$ $\rightarrow x = -2$ $x = 7, x = -2$	۲
۹) $A = \{1, 2, 3\} \rightarrow A \times B = \{(1,2), (1,3), (2,2), (2,3), (3,2), (3,3)\}$ $B = \{2, 3\} \rightarrow B^c = \{(1,2), (1,3), (2,2), (2,3)\}$ $\rightarrow A \times B - B^c = \{(1,2), (1,3)\}$ $\rightarrow A \times B - B^c = \{(1,2), (1,3)\}$				۳
۱۰) $g(x) = \{(3, m^2), (2, 1), (m, 4), (3, m+2), (-2, m), (-1, 3)\}$ $m^2 = m+2 \rightarrow m^2 - m - 2 = 0 \rightarrow (m-2)(m+1) = 0$ $\rightarrow m = 2 \vee m = -1$ $m = 2 \rightarrow \{(3, 4), (2, 1), (3, 4), (-3, 2), (-2, 2), (3, 4)\}$ $m = -1 \rightarrow \{(3, 1), (2, 1), (-3, 4), (3, 1), (-2, -1), (-1, 3)\}$ $\rightarrow m = -1$		۱۱) $f(x) = \{(3, m^2), (2, 1), (-3, m), (-2, m), (3, m+2), (m, 4)\}$ $m^2 = m+2 \rightarrow m^2 - m - 2 = 0 \rightarrow (m-2)(m+1) = 0$ $\rightarrow m = 2 \vee m = -1$ $m = 2 \rightarrow \{(3, 4), (2, 1), (-3, 2), (-2, 2), (3, 4), (2, 4)\}$ $m = -1 \rightarrow \{(3, 1), (2, 1), (-3, -1), (-2, -1), (3, 1), (-1, 4)\}$ $\rightarrow m = -1$		۴
۱۲) $f(x) = \{(1, a^2), (1, 3a-2), (a^2, 3a+2)\}$ $a^2 = 3a-2 \rightarrow a^2 - 3a + 2 = 0 \rightarrow (a-2)(a-1) = 0 \rightarrow a = 2 \vee a = 1$ $a = 1 \rightarrow f(x) = \{(1, 1), (1, 1), (1, 5)\} \rightarrow$ تکرار نیست! $a = 2 \rightarrow f(x) = \{(1, 4), (1, 4), (4, 8)\} \rightarrow$ تکرار هست! $\rightarrow a = 2$				۵

$$f(x) = \begin{cases} x^2 - 1 & x \leq 1 \\ ax + b & 1 \leq x \leq 2 \\ x^2 & x \geq 2 \end{cases}$$

$$x = 1 \rightarrow 1^2 - 1 = a \cdot 1 + b \rightarrow 1 - 1 = a + b \rightarrow a + b = 0$$

$$x = 2 \rightarrow 2^2 = a \cdot 2 + b \rightarrow 4 = 2a + b$$

$$\begin{aligned} 2a + b &= 4 \\ -a - b &= 0 \\ \hline a &= -4 \end{aligned}$$

$$a + b = 0 \rightarrow b = -a$$

$$a \cdot b = -4 \times (-1) = -4$$

$$\rightarrow a \cdot b = \boxed{-4}$$

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$$f(x) = \begin{cases} 2x - 3 & x \geq K \\ 4 - 3x & x \leq K \end{cases}$$

$$\rightarrow x = K$$

$$2K - 3 = 4 - 3K$$

$$5K = 7$$

$$\rightarrow K = \frac{7}{5} \rightarrow K = \boxed{\frac{7}{5}}$$

7

$$a) y^2 + x^2 + 2x + 1 = 0$$

$$y^2 = -x^2 - 2x - 1$$

$$y^2 = -(x^2 + 2x + 1)$$

$$y^2 = -(x + 1)^2$$

$$\rightarrow y = 0 \rightarrow x + 1 = 0 \rightarrow x = -1$$

! پاسخ هست

$$c) |y| = \sqrt{1 - \cos x}$$

$$|y| = \sqrt{1 - \cos 180^\circ}$$

$$|y| = \sqrt{1 - (-1)} = \sqrt{1 + 1} = \sqrt{2}$$

$$\rightarrow y = \pm \sqrt{2}$$

! پاسخ نیست

$$b) |x| + |y| = 3$$

$$x = 0 \rightarrow |y| = 3$$

$$\rightarrow y = \pm 3$$

! پاسخ نیست

$$d) y^3 - 4y = x^2 - 4x + 1$$

$$x = 3 \rightarrow y(y^2 - 4) = 9 - 12 + 1$$

$$y(y^2 - 4) = 0$$

$$\rightarrow y = 0$$

$$\rightarrow y = \pm \sqrt{4}$$

! پاسخ نیست

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$$b) y^3 + 2y = 5x$$

$$\begin{cases} 5x_1 = y_1^3 + 2y_1 \\ 5x_2 = y_2^3 + 2y_2 \end{cases} \xrightarrow{x_1 = x_2} y_1^3 - y_2^3 = -2(y_1 - y_2)$$

$$\rightarrow (y_1 - y_2)(y_1^2 + y_1 y_2 + y_2^2) = -2(y_1 - y_2)$$

$$(y_1 - y_2)(y_1^2 + y_1 y_2 + y_2^2 + 2) = 0 \rightarrow y_1 = y_2$$

$$x^2 + y_2 x + y_2^2 + 2 \rightarrow y_2^3 - 4x_1(y_2^2 + 2) =$$

$$y_2^3 - 4y_2^2 - 8 = -16y_2^2 - 8 = -(16y_2^2 + 8) \rightarrow \Delta < 0 \rightarrow \text{پاسخ ندارد!}$$

! پاسخ هست

$$d) \sqrt[3]{y} + x^2 = 2x - 2$$

$$\begin{aligned} -x_1^2 + 2x_1 - 2 &= \sqrt[3]{y_1} \\ -x_2^2 + 2x_2 - 2 &= \sqrt[3]{y_2} \end{aligned} \xrightarrow{x_1 = x_2} \sqrt[3]{y_1} = \sqrt[3]{y_2}$$

$$\rightarrow \sqrt[3]{y_1} = \sqrt[3]{y_2} \rightarrow y_1 = y_2$$

! پاسخ هست

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$$b) \sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} = 2$$

$$a + \frac{1}{a} = 2 \rightarrow a = 1$$

$$\sqrt{\frac{x}{y}} = 1 \rightarrow \frac{x}{y} = 1 \rightarrow x = y$$

! پاسخ هست

$$d) x \sin y = y \sin x$$

$$\rightarrow x = \{K\pi\} = \{\pi, 2\pi, \dots\}$$

$$\pi \times \sin y = y \times 0 \rightarrow \pi \cdot \sin y = 0$$

$$\rightarrow \sin y = 0 \rightarrow y = K\pi \rightarrow \{0, \pi, 2\pi, \dots\}$$

! پاسخ نیست

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