

نام و نام خانوادگی تاریخ پاسخنامه تشریحی تکلیف شماره کلاس نام و نام خانوادگی تاریخ

الف) $2x^2 - 8x - 6 = 0$ $x^2 - 4x - 3 = 0$ $(x-9)(x+1) = 0$ $x = \frac{9}{2} \pm \frac{\sqrt{13}}{2}$ $x = \frac{9}{2} \pm \frac{\sqrt{13}}{2}$
 ب) $2x^2 - 8x - 3 = 0$ $x^2 - 4x - 1.5 = 0$ $(x-9)(x+1) = 0$ $x = \frac{9}{2} \pm \frac{\sqrt{13}}{2}$ $x = \frac{9}{2} \pm \frac{\sqrt{13}}{2}$
 ج) $5x^2 - 4x - 3 = 0$ $x^2 - 0.8x - 0.6 = 0$ $(x-4)(x+2) = 0$ $x = \frac{4}{5} \pm \frac{\sqrt{13}}{5}$ $x = \frac{4}{5} \pm \frac{\sqrt{13}}{5}$
 د) $2x^2 - 8x - 11 = 0$ $x^2 - 4x - 5.5 = 0$ $(x-1)(x+3) = 0$ $x = \frac{1}{2} \pm \frac{\sqrt{13}}{2}$ $x = \frac{1}{2} \pm \frac{\sqrt{13}}{2}$

الف) $x^2 - 8x - 14 = 0$ $(x-7)(x+2) = 0$ $x = 7$ $x = -2$
 ب) $2x^2 + 8x + 3 = 0$ $a + c = b \Rightarrow x = -1$ $x = -\frac{3}{2}$
 ج) $x^2 + 2x + 3 = 0$ $x = \frac{-2 \pm \sqrt{4-12}}{2} \Rightarrow \Delta < 0$ $\Delta < 0$
 د) $3x^2 + 3x - 2 = 0$ $x = \frac{-3 \pm \sqrt{9+24}}{6} \Rightarrow \frac{-1}{2} \pm \frac{\sqrt{33}}{6}$

$A = \{1, 2, 3\}$ $A \times B = B^2 = \{(1, 2)(1, 3)\}^2$
 $B = \{2, 3\}$
 $A \times B = \{(1, 2)(1, 3)(2, 2)(2, 3)(3, 2)(3, 3)\}$
 $B \times B = \{(2, 2)(2, 3)(3, 2)(3, 3)\}$

الف) $f(x) = \{(3, m^2)(2, 1)(-3, m)(-2, m)(3, m+2)(m, 4)\}$
 $m^2 = m+2$ $m^2 - m - 2 = 0$ $m = -1$ $m = 2$
 $(2, 1)(2, 4)$

ب) $g(x) = \{(3, m^2)(2, 1)(m, 4)(3, m+2)(-2, m)(-1, 3)\}$
 $m^2 = m+2$ $m = -1$ $m = 2 \rightarrow (2, 1)(2, 4)$
 برای هیچ مقدار از m معادله صحیح نیست.

$f(x) = \{(1, a^2)(1, 3a-2)(a^2, 3a+2)\}$
 $a^2 = 3a-2$ $a^2 - 3a + 2 = 0$ $a = 1$ $a = 2$
 $(1, 1)(1, 1)$ $\{(1, 4)(4, 1)\}$

$$f(x) = \begin{cases} x^2 - 1 & x \leq 1 \\ ax + b & 1 \leq x \leq 2 \\ x^2 & x > 2 \end{cases}$$

$$\begin{aligned} x=1 \Rightarrow x^2 - 1 &= ax + b = 0 \\ a + b &= 0 \\ x=2 \Rightarrow 2a + b &= 2^2 = 4 \\ a + b &= 0 \\ \begin{cases} 2a + b = 4 \\ a + b = 0 \end{cases} \Rightarrow a &= 1 \\ b &= -1 \\ ax + b &= \underline{x - 1} \end{aligned}$$

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$$f(x) = \begin{cases} x^2 - 3 & x \geq k \\ 5 - 3x & x \leq k \end{cases}$$

$$\begin{aligned} x=k \Rightarrow k^2 - 3 &= 5 - 3k \\ 8k &= 8 \Rightarrow k &= 1 \end{aligned}$$

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$$\text{مثال نقض: } \left| \frac{1}{0} \right|, \left| \frac{1}{\sqrt{2}} \right|, \left| \frac{1}{\sqrt{2}} \right|$$

الف) $y^3 - 4y = x^2 - 4x + 3 \times$ $\begin{cases} (x_1, y_1) \in f \\ (x_2, y_2) \in f \end{cases} \Rightarrow \begin{cases} x_1 = x_2 \\ y_1 = y_2 \end{cases}$ تابع نیست

$y_1^3 - 4y_1 = y_2^3 - 4y_2$

$y_1^3 - 4y_1 - y_2^3 + 4y_2 = (y_1 - y_2)(y_1^2 + y_1y_2 + y_2^2 - 4)$

$\Rightarrow |x_1| + |y_1| = 3 \times$ $x=1$ $|y|=2$ $y=2$ $y=-2$

ج) $|y| = \sqrt{1 - \cos x}$ $-1 < \cos x \leq 1$ $x = \frac{\pi}{2} \Rightarrow \cos x = 0 \Rightarrow y = 1$ $y = -1$

د) $y^2 + x^2 + 2x + 1 = 0$ $\sqrt{y^2 + (x+1)^2} = 0 \Rightarrow$ نقطه $(-1, 0)$ تابع است

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الف) $\sqrt[3]{y} + x^2 = 2x - 2$ $\begin{cases} (x_1, y_1) \in f \\ (x_2, y_2) \in f \end{cases} \Rightarrow \begin{cases} x_1 = x_2 \\ y_1 = y_2 \end{cases}$ تابع است

$\sqrt[3]{y_1} = -x_1^2 + 2x_1 - 2$ $\sqrt[3]{y_2} = -x_2^2 + 2x_2 - 2$

$\Rightarrow \sqrt[3]{y_1} = \sqrt[3]{y_2} \Rightarrow y_1 = y_2$

$\Rightarrow y_1^3 + 2y_1 = y_2^3 + 2y_2$

$\Rightarrow y_1^3 - y_2^3 + 2y_1 - 2y_2 = 0$ $(y_1 - y_2)(y_1^2 + y_1y_2 + y_2^2 + 2)$

$\Rightarrow y_1 = y_2$ تابع است

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الف) $x \sin y = y \sin x$ $x=0 \Rightarrow y \in \mathbb{R} \Rightarrow$ تابع نیست

مثال نقض: $y=0$ $x \neq 0$

$\Rightarrow \sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} = 2$ تابع است

$a = \sqrt{\frac{x}{y}}$ $a + \frac{1}{a} = 2$ $a^2 + 1 = 2a$ $a^2 - 2a + 1 = 0$ $(a-1)^2 = 0$ $a = 1$ $\sqrt{\frac{x}{y}} = 1$

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