

الف) $4x^2 - 5x - 4 = 0 \rightarrow x^2 - 5x - 4 = 0 \rightarrow (x-9)(x+4) \Rightarrow \begin{cases} x = \frac{9}{4} = \frac{3}{1} \\ x = -\frac{4}{4} = -\frac{2}{1} \end{cases}$

ب) $3x^2 - 1x - 3 = 0 \rightarrow x^2 - 1x - 9 = 0 \rightarrow (x+1)(x-9) \Rightarrow \begin{cases} x = -\frac{1}{3} \\ x = \frac{9}{3} = 3 \end{cases}$

ج) $4x^2 - 5x - 3 = 0 \rightarrow x^2 - 5x - 12 = 0 \rightarrow (x-4)(x+3) \Rightarrow \begin{cases} x = \frac{4}{2} = \frac{2}{1} \\ x = -\frac{3}{2} = -\frac{1}{1} \end{cases}$

د) $2x^2 - 5x - 12 = 0 \rightarrow x^2 - 5x - 24 = 0 \rightarrow (x-8)(x+3) \Rightarrow \begin{cases} x = \frac{8}{4} = 2 \\ x = -\frac{3}{2} \end{cases}$

الف) $x^2 - 5x - 12 = 0 \xrightarrow{\text{جد مشترک}} (x+2)(x-7) \Rightarrow \begin{cases} x = -2 \\ x = 7 \end{cases}$

ب) $2x^2 + 5x + 3 = 0 \xrightarrow{a+c=b} \begin{cases} x = -1 \\ x = -\frac{3}{2} \end{cases}$

ج) $x^2 + 2x + 3 = 0 \xrightarrow{\text{ریشه}} \Delta \Rightarrow 4 - 4(1)(3) = -8 \rightarrow \Delta < 0$

د) $3x^2 + 3x - 2 = 0 \xrightarrow{\text{دلتا}} \Delta = 9 - 4(3)(-2) = 33 \Rightarrow \begin{cases} x = \frac{-3 + \sqrt{33}}{6} \\ x = \frac{-3 - \sqrt{33}}{6} \end{cases}$

$A = \{1, 2, 3\}, B = \{2, 3\} \quad A \times B - B^2 = ?$

$A \times B = \{(1, 2), (1, 3), (2, 2), (2, 3), (3, 2), (3, 3)\}$, $B \times B = \{(2, 2), (2, 3), (3, 2), (3, 3)\}$

$A \times B - B^2 = \{(1, 2), (1, 3)\}$

الف) $f(n) = \{(3, m^2), (2, 1), (-3, m), (-2, m), (3, m+2), (m, 4)\}$

$m^2 = m+2 \rightarrow m^2 - m - 2 = 0 \xrightarrow{a+c=b} \begin{cases} m = -1 \\ m = 2 \end{cases}$

ب) $g(n) = \{(3, m^2), (2, 1), (m, 4), (3, m+2), (-2, m), (-1, 3)\}$

$m^2 = m+2 \rightarrow m^2 - m - 2 = 0 \xrightarrow{a+c=b} \begin{cases} m = -1 \\ m = 2 \end{cases}$

پس دو به عنوان به ازای $m = -1$ بررسی می‌کنیم $f(n)$ و $g(n)$ تابع منسوب

الف) $f(x) = \{(1, a^2), (1, 3a-2), (a^2, 3a+2)\} \rightarrow \text{تابع منسوب}$

ب) $f(x) = \{(1, 4), (1, 4), (3, 8)\} \rightarrow \text{تابع منسوب}$

$$a \times b = ?$$

$$f(n) = \begin{cases} n^2 - 1 & n \leq 1 \\ a \times b & 1 < n \leq 2 \\ n^3 & n > 2 \end{cases} \rightarrow \begin{matrix} n=1 \rightarrow (1)^2 - 1 = a+b \Rightarrow a+b=0 \rightarrow \boxed{a=-b} \\ n=2 \rightarrow 2a+b=1 \xrightarrow{a=-b} -2b+b=1 \rightarrow -b=1 \rightarrow \boxed{b=-1} \\ \Rightarrow b=-1 \rightarrow 2a-1=1 \rightarrow 2a=2 \rightarrow \boxed{a=1} \end{matrix}$$

$$a \times b = -1 \times 1 = \boxed{-1}$$

4

$$f(n) = \begin{cases} 2n-3 & n > K \\ 5-3n & n \leq K \end{cases} \rightarrow n=K \rightarrow 2K-3 = 5-3K \rightarrow 5K=8 \rightarrow \boxed{K=\frac{8}{5}}$$

V

(1) نکته: جزو = تابع نیست

الف) $y^3 - 4y = x^2 - 5x + 3 \rightarrow y^3 - y = 0 \rightarrow y(y^2 - 1) = 0 \rightarrow y = 0, \pm 1$

ب) $|x| + |y| = 3 \rightarrow \text{if } (x=0) \rightarrow |y| = 3 \rightarrow y = \pm 3$

ج) $|y| = \sqrt{1 - \cos x} \rightarrow 1 - \cos x \geq 0 \rightarrow \cos x \leq 1$

if $(\cos x = 0) \Rightarrow |y| = 1 \rightarrow y = \pm 1$

د) $y^2 + x^2 + 2x + 1 = 0 \rightarrow y^2 + (x+1)^2 = 0$

$y^2 = 0 \rightarrow \boxed{y=0}, x+1=0 \rightarrow \boxed{x=-1}$

چیز دایره است که مرکز آن در نقطه (-1, 0) است و شعاع آن 1 است.

الف) $\sqrt[3]{y} + x^2 = 2n+2 \rightarrow \begin{cases} \sqrt[3]{y_1} = -x_1^2 + 2n_1 + 2 \\ \sqrt[3]{y_2} = -x_2^2 + 2n_2 + 2 \end{cases} \xrightarrow{n_1=n_2} \sqrt[3]{y_1} = \sqrt[3]{y_2} \rightarrow \boxed{y_1=y_2}$

ب) $y^3 + 2y = 5n \rightarrow \begin{cases} y_1^3 + 2y_1 = 5n_1 \\ y_2^3 + 2y_2 = 5n_2 \end{cases} \xrightarrow{n_1=n_2} y_1^3 + 2y_1 = y_2^3 + 2y_2 \rightarrow y_1^3 - y_2^3 + 2y_1 - 2y_2 = 0$

$(y_1 - y_2)(y_1^2 + y_1 y_2 + y_2^2) + 2(y_1 - y_2) = 0 \Rightarrow (y_1 - y_2)(y_1^2 + y_1 y_2 + y_2^2 + 2) = 0 \rightarrow y_1 = y_2 \rightarrow \boxed{y_1=y_2}$

الف) $x \sin y = y \sin x \text{ if } (x=1) \Rightarrow \sin y = 0 \rightarrow y = 0, \pi$

ب) $\left(\sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}}\right)^2 = 2 \rightarrow \frac{x}{y} + \frac{y}{x} + 2\sqrt{\frac{xy}{xy}} = 2 \rightarrow \frac{x^2 + y^2}{xy} + 2 = 2$

$\frac{x^2 + y^2}{xy} - 2 = 0 \xrightarrow{\times xy} x^2 + y^2 - 2xy = 0 \rightarrow (x-y)^2 = 0 \rightarrow \boxed{x=y}$

10