

۲. افرینا فیلی خوب نوشتار!

۱) $4n^2 - 5n - 4 = 0 \rightarrow n^2 - 5n - 4 = 0 \rightarrow (n-9)(n+4) = 0 \rightarrow n = 9 = \frac{9}{1} \quad \checkmark$
 $n = -\frac{4}{1} = -4 \quad \checkmark$

۲) $3n^2 - 8n - 9 = 0 \rightarrow n^2 - 8n - 9 = 0 \rightarrow (n-9)(n+1) = 0 \rightarrow n = 9 = \frac{9}{1} \quad \checkmark$
 $n = -\frac{1}{1} = -1 \quad \checkmark$

۳) $4n^2 - 4n - 12 = 0 \rightarrow n^2 - 4n - 12 = 0 \rightarrow (n-6)(n+2) = 0 \rightarrow n = \frac{6}{1} = 6 \quad \checkmark$
 $n = -\frac{2}{1} = -2 \quad \checkmark$

۴) $2n^2 - 4n - 12 = 0 \rightarrow n^2 - 4n - 24 = 0 \rightarrow (n-8)(n+3) = 0 \rightarrow n = \frac{8}{1} = 8 \quad \checkmark$
 $n = -\frac{3}{1} = -3 \quad \checkmark$

۵) $n^2 - 5n - 14 = 0 \rightarrow (n-7)(n+2) = 0 \rightarrow n = 7 \quad \checkmark$
 $n = -2 \quad \checkmark$

۶) $2n^2 + 4n + 2 = 0 \rightarrow n^2 + 2n + 1 = 0 \rightarrow (n+1)(n+1) = 0 \rightarrow n = -\frac{1}{1} = -1 \quad \checkmark$
 $n = -\frac{1}{1} = -1 \quad \checkmark$

۷) $n^2 + 2n + 3 = 0 \quad \Delta = 4 - 12 = -8 \rightarrow \Delta < 0 \rightarrow$ ~~no solution~~

۸) $2n^2 + 3n - 2 = 0 \rightarrow \Delta = 9 + 16 = 25 \rightarrow \Delta > 0 \rightarrow n = \frac{-3 \pm \sqrt{25}}{4}$

$A = \{1, 2, 3\} \quad B = \{2, 3\} \quad A \times B = \{(1, 2), (1, 3), (2, 2), (2, 3), (3, 2), (3, 3)\}$
 $B^c = \{(1, 2), (1, 3)\}$

۹) $f(n) = \{(2, m^2), (2, 1), (-3, m), (-2, m), (3, m+2), (m, 4)\}$

$m^2 = m + 2 \rightarrow m^2 - m - 2 = 0 \rightarrow (m-2)(m+1) = 0 \rightarrow m = 2 \rightarrow \{(2, 4), (2, 1), (-3, 2), (-2, 2), (3, 4), (2, 4)\} \times$
 $m = -1 \rightarrow \{(2, 1), (2, 1), (-3, -1), (-2, -1), (3, -1), (-1, 4)\} \checkmark$

۱۰) $g(n) = \{(2, m^2), (2, 1), (m, 4), (3, m+2), (-2, m), (-1, 3)\}$

$m^2 = m + 2 \rightarrow m^2 - m - 2 = 0 \rightarrow m = 2 \rightarrow \{(2, 4), (2, 1), (2, 4), \dots\} \times$

$m = -1 \rightarrow \{(2, 1), (2, 1), (-1, 4), (3, 1), (-2, -1), (-1, 3)\} \times$

هیچ جوابی برای m وجود ندارد.

$$f(n) = (1, n^2) (1, 5n-2) (n^2, 4n+2)$$

④

$$n^2 - 5n + 2 = 0 \rightarrow (n-2)(n-1) = 0 \rightarrow n=2 \quad n=1$$

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$$n=2 \rightarrow \{(1, 4) (1, 8) (4, 1)\} \checkmark \quad n=1 \rightarrow \{(1, 1) (1, 1) (1, 1)\} \times$$

$$f(n) = \begin{cases} n^2 - 1 & n \leq 1 \\ an + b & 1 \leq n \leq 2 \\ n^2 & n \geq 2 \end{cases}$$

$$n=1 \rightarrow 1^2 - 1 = a + b \rightarrow a = -b$$

$$n=2 \rightarrow 2a + b = 1 \rightarrow 2a - a = 1 \rightarrow a = 1 \rightarrow b = -1$$

$$\rightarrow a \times b = 1 \times -1 = -1$$

⑦

$$f(n) = \begin{cases} kn + 2 & n \geq k \\ 2 - kn & n \leq k \end{cases}$$

$$n=k \rightarrow 2k + 2 = 2 - 2k \rightarrow 4k = 0 \rightarrow k=0$$

$$k = \frac{2}{4} = \frac{1}{2}$$

$$k = \frac{1}{2}$$

⑧

$$y^5 - 4y = n^2 - 4n + 2 \quad n=1 \rightarrow y^5 - 4y = 0 \rightarrow y(y^4 - 4) = 0 \rightarrow y(y - \sqrt{2})(y + \sqrt{2}) = 0$$

$$\rightarrow y=0 \quad y=\sqrt{2} \quad y=-\sqrt{2} \rightarrow \text{three solutions}$$

$$\Rightarrow |n| + |y| = 2 \quad n=0 \rightarrow |y| = 2 \rightarrow y = \pm 2 \rightarrow \text{two solutions}$$

$$y) |y| = \sqrt{1 - \cos n} \quad n=\pi \rightarrow |y| = \sqrt{1 - (-1)} = \sqrt{2} \rightarrow |y| = 1 \rightarrow y = \pm 1 \rightarrow \text{two solutions}$$

$$y) y^2 + n^2 + 2n + 1 = 0 \rightarrow y^2 + (n+1)^2 = 0 \rightarrow y=0, n=-1 \rightarrow \text{one solution}$$

$$z) \sqrt[3]{y} + n^2 = 2n - 2 \rightarrow y = (-n^2 + 2n - 2)^3 \rightarrow y_1 = (-n^2 + 2n - 2)^3 \rightarrow y_2 = (-n^2 + 2n - 2)^3 \rightarrow y_3 = (-n^2 + 2n - 2)^3$$

⑨

$$\div) \begin{cases} y_1' + y_2' = \sin x \\ y_1' + y_2' = \sin x \end{cases} \rightarrow y_1' + y_2' = \sin x \rightarrow y_1' + y_2' = \sin x$$

$$\rightarrow (y_1 - y_2)(y_1' + y_2' + y_1 y_2) + y_2(y_1 - y_2) = 0 \rightarrow (y_1 - y_2)(y_1' + y_2' + y_1 y_2 + y_2) = 0$$

صفر است

توجه: اگرچه سینوس مرتبه اول است

$$\sin x = y \sin x \rightarrow x = 0 \rightarrow y \sin 0 = 0$$

توجه: (5)

$$\div) \sqrt{\frac{x}{y} + \sqrt{\frac{y}{x}}} = 1 \rightarrow \frac{x}{y} + \frac{y}{x} + 1 = \sqrt{\frac{xy}{xy}} = 1 \rightarrow \frac{x^2 + y^2}{xy} = 1$$

$$\rightarrow x^2 + y^2 - xy = 0 \rightarrow (x - y)^2 = 0 \rightarrow x = y$$

چون برای هر مقدار x و y داریم $(x \neq 0, y \neq 0)$ دارد نه!

(۱۰-ب) $\sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} = 2$

این عبارت تنها زمانی برقرار است که $1 = \sqrt{\frac{x}{y}}$ باشد پس

$\sqrt{\frac{x}{y}} = 1 \leadsto \frac{x}{y} = 1 \leadsto x = y$ و $(x, y \neq 0)$

