

الف)  $x^2 - 5x - 6 = 0 \rightarrow x^2 - 5x - 6 = (x-6)(x+1) = 0 \rightarrow \begin{cases} x_1 = \frac{6}{1} = 6 \\ x_2 = \frac{-1}{1} = -1 \end{cases}$

ب)  $x^2 - 1x - 2 = 0 \rightarrow x^2 - 1x - 2 = (x-2)(x+1) = 0 \rightarrow \begin{cases} x_1 = \frac{2}{1} = 2 \\ x_2 = \frac{-1}{1} = -1 \end{cases}$

ج)  $x^2 - 4x - 12 = 0 \rightarrow x^2 - 4x - 12 = (x-6)(x+2) = 0 \rightarrow \begin{cases} x_1 = \frac{6}{1} = 6 \\ x_2 = \frac{-2}{1} = -2 \end{cases}$

د)  $x^2 - 5x - 12 = 0 \rightarrow x^2 - 5x - 12 = (x-8)(x+3) = 0 \rightarrow \begin{cases} x_1 = \frac{8}{1} = 8 \\ x_2 = \frac{-3}{1} = -3 \end{cases}$

الف)  $x^2 - 5x - 12 = (x-8)(x+3) = 0 \rightarrow x_1 = 8, x_2 = -3$

ب)  $2x^2 + 5x + 3 = 0 \xrightarrow{a+c=b} x_1 = -1, x_2 = -\frac{3}{2}$

ج)  $x^2 + 2x + 3 = 0 \rightarrow \Delta = b^2 - 4ac = 4 - 12 = -8 < 0 \rightarrow$  ریشه حقیقی ندارد

د)  $3x^2 + 2x - 2 = 0 \rightarrow x = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-2 \pm \sqrt{4 + 24}}{6} \rightarrow x_1 = \frac{-2 + \sqrt{28}}{6}, x_2 = \frac{-2 - \sqrt{28}}{6}$

$A = \{1, 2, 3\} \quad B = \{2, 3\}$

$A \times B = \{(1, 2), (1, 3), (2, 2), (2, 3), (3, 2), (3, 3)\} \quad B^c = \{(2, 2), (2, 3), (3, 2), (3, 3)\}$

$A \times B - B^c = \{(1, 2), (1, 3)\}$

الف)  $f(x) = \{(3, m^2), (2, 1), (-3, m), (-2, m), (3, m+2), (m, 4)\}$

$m^2 = m + 2 \rightarrow m^2 - m - 2 = (m-2)(m+1) = 0 \rightarrow m_1 = 2, m_2 = -1$

ب)  $g(x) = \{(3, m^2), (2, 1), (m, 4), (3, m+2), (-2, m), (-1, 3)\}$

$m^2 = m + 2 \rightarrow m_1 = 2, m_2 = -1$

$$f(x) = \{(1, a^2), (1, 3a-2), (a^2, 3a+2)\}$$

$$a^2 = 3a-2 \rightarrow a^2 - 3a + 2 = 0 \xrightarrow{(\text{عقبات})} a+b+c=0 \rightarrow a_1=1, a_2=2$$

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$$f(x) = \begin{cases} x^2-1 & x \leq 1 \\ ax+b & 1 \leq x \leq 2 \\ x^2 & x \geq 2 \end{cases}$$

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$$\left. \begin{array}{l} \text{if } x=1 \rightarrow 1-1 = a+b \Rightarrow a+b=0 \\ \text{if } x=2 \rightarrow 2^2 = 2a+b \Rightarrow 2a+b=4 \end{array} \right\} a=1, b=-1 \rightarrow a \times b = -4$$

$$f(x) = \begin{cases} 2x-3 & x \geq k \\ \varepsilon-3x & x \leq k \end{cases}$$

$$2x-3 = \varepsilon-3x \rightarrow 5x = \varepsilon \rightarrow x = \frac{\varepsilon}{5} = k$$

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چون  $y$  در  $y$  با  $x$  در  $x$  است  $\rightarrow$  تابع نیست  $\times$   $y^3 - 9y = x^2 - 4x + 3$  الف

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ب)  $|x| + |y| = 3$   $\times$  تابع نیست  $\rightarrow$  مثال حقیقی:  $x=1 \rightarrow y = \pm 2$

ج)  $|y| = \sqrt{1 - \cos x}$   $\times$  تابع نیست  $\rightarrow$  مثال حقیقی:  $x=90^\circ \rightarrow \cos x = 0 \rightarrow y = \pm 1$

د)  $y^2 + x^2 + 2x + 1 = 0 \rightarrow y^2 + (x+1)^2 = 0$  به انلی اعداد حقیقی، سمت چپ همیشه  $\geq 0$  است.  $\rightarrow$  تابع است.  $(-1, 0)$  تابع یک نقطه  $\rightarrow$  تابع است  $\checkmark$

الف)  $\sqrt[3]{y} + x^2 = 2x-2 \rightarrow \sqrt[3]{y} = -x^2 + 2x-2$

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$$\left. \begin{array}{l} \sqrt[3]{y_1} = -x_1^2 + 2x_1 - 2 \\ \sqrt[3]{y_2} = -x_2^2 + 2x_2 - 2 \end{array} \right\} x_1 = x_2 \rightarrow \sqrt[3]{y_1} = \sqrt[3]{y_2} \rightarrow y_1 = y_2 \quad \checkmark \text{ تابع است}$$

ب)  $y^2 + 2y = 5x$

$$\left. \begin{array}{l} y_1^2 + 2y_1 = 5x_1 \\ y_2^2 + 2y_2 = 5x_2 \end{array} \right\} x_1 = x_2 \rightarrow y_1^2 + 2y_1 = y_2^2 + 2y_2 \rightarrow y_1^2 - y_2^2 = 2y_2 - 2y_1$$

$$\rightarrow (y_1 - y_2)(y_1^2 + y_1y_2 + y_2^2) + 2(y_1 - y_2) = 0 \rightarrow (y_1 - y_2)(y_1^2 + y_1y_2 + y_2^2 + 2) = 0$$

$$\rightarrow \left\{ \begin{array}{l} y_1 - y_2 = 0 \rightarrow y_1 = y_2 \\ y_1^2 + y_1(y_2^2 + 2) + y_2^2 + 2 = 0 \rightarrow \Delta = y_2^2 - 4(y_2^2 + 2) = y_2^2 - 4y_2^2 - 8 = -3y_2^2 - 8 < 0 \end{array} \right. \times$$

$y$  هر عددی می تواند باشد  $\rightarrow$  if  $x=0$  ✓ تابع نیست  $x \sin y = y \sin x$  (الف)

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$$\rightarrow \sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} = 2 \quad \text{if } \sqrt{\frac{x}{y}} = a \quad \rightarrow a + \frac{1}{a} = 2 \rightarrow a^2 - 2a + 1 = 0$$

اگر  $x$  و  $y$  با هم منفی باشند باز هم جواب درست است

$$\rightarrow (a-1)^2 = 0 \rightarrow a = 1$$

$$\rightarrow \sqrt{\frac{x}{y}} = 1 \rightarrow \frac{x}{y} = 1 \rightarrow x = y \quad \checkmark \text{ تابع است}$$

به ازای  $(x > 0)$  فقط یک  $y$  داریم

$$(10-ب) \sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} = 2$$

این عبارت تنها زمانی برقرار است که  $\sqrt{\frac{x}{y}} = 1$  باشد پس

$$\sqrt{\frac{x}{y}} = 1 \leadsto \frac{x}{y} = 1 \leadsto x = y \text{ و } (x, y \neq 0)$$

