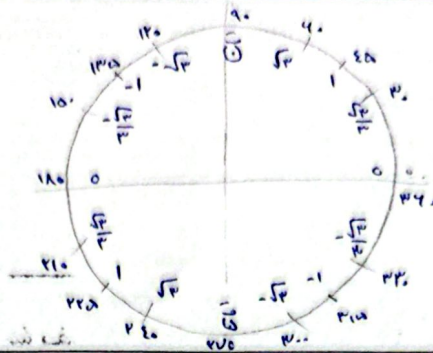




1

الف) $\sin \frac{\pi}{4} + \cos \pi + 2 \tan \frac{3\pi}{4} = \frac{1}{\sqrt{2}} + (-1) + 2(-1) = \frac{1-2-2}{\sqrt{2}} = \frac{-3}{\sqrt{2}}$

ب) $-\cos \frac{5\pi}{4} + \sin \frac{2\pi}{3} - 3 \sin \frac{\pi}{6} = -(-\frac{\sqrt{2}}{2}) + \frac{\sqrt{3}}{2} - 3(\frac{1}{2}) = \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} - \frac{3}{2} = \frac{\sqrt{2} + \sqrt{3} - 3}{2}$

ج) $\cot \frac{2\pi}{3} + 3 \cot \frac{\pi}{3} - 2 \tan \frac{5\pi}{4} = -\frac{\sqrt{3}}{3} + 3(\frac{1}{\sqrt{3}}) - 2(-\frac{\sqrt{2}}{2}) = -\frac{\sqrt{3}}{3} + \sqrt{3} + \sqrt{2} = \frac{2\sqrt{3}}{3} + \sqrt{2}$

2

د) $\cos(\sin \pi) + \sin(\pi \cos \frac{\pi}{2}) = \cos(0) + \sin(\pi \cdot 0) = 1 + 0 = 1$

3

$$\begin{cases} x^2 + \varepsilon x & n \gg a \\ x^2 + a + 2 & n \leq a \end{cases} \Rightarrow a^2 + \varepsilon a = 2a + a + 2 \rightarrow a^2 + a - 2 = 0 \xrightarrow{a+b+c=0} \boxed{a=1}, a=-2$$

$f(1) = 2(1) + 1 + 2 = 5$

4

$$f(n) = \begin{cases} 2n^2 - b & |x+1| \geq 2 \\ a + 2x & -2 \leq x \leq -1 \end{cases} \xrightarrow{\text{بنا بر درستی شرط}} n = -3 \rightarrow 18 - b = a - 4 \rightarrow a + b = 22$$

5

الف) $y = \frac{1}{x^3 - \varepsilon x^2} \rightarrow x^3 - \varepsilon x^2 \neq 0 \rightarrow x^2(x - \varepsilon) \neq 0 \rightarrow x \neq 0, \varepsilon \rightarrow D_f = \mathbb{R} - \{0, \varepsilon\}$

ب) $y = \frac{1}{2x^2 + 4x^2 + 2x + 2} = \frac{1}{2x(x^2 + 2x + 1) + 2} = \frac{1}{2x(x+1)^2 + 2}$

$\rightarrow 2x(x+1)^2 \neq -2 \rightarrow x \neq -2 \rightarrow D_f = \mathbb{R} - \{-2\}$

الف) $\sqrt{x^2-1} + \sqrt{x-1} \rightarrow x^2-1 \geq 0 \rightarrow (x-1)(x+1) \geq 0 \rightarrow \frac{1}{x-1} \rightarrow (-\infty, -1] \cup (1, +\infty)$ $\hookrightarrow x-1 \geq 0 \rightarrow x \geq 1 \Rightarrow D_f = [1, +\infty)$ ب) $\frac{\sqrt{4-3x}}{\sqrt{4x-3}} \rightarrow 4-3x \geq 0 \rightarrow 3x \leq 4 \rightarrow x \leq \frac{4}{3}$ $\hookrightarrow 4x-3 \geq 0 \rightarrow 4x \geq 3 \rightarrow x \geq \frac{3}{4} \Rightarrow D_f = (\frac{3}{4}, \frac{4}{3}]$	6
$4x-a \geq 0 \rightarrow a \leq 4x \rightarrow x \geq \frac{a}{4}$ $b-3x \geq 0 \rightarrow 3x \leq b \rightarrow x \leq \frac{b}{3}$ $\frac{a}{4} \leq x \leq \frac{b}{3} \xrightarrow{D_f = \{x\}} \frac{a}{4} = 1 \rightarrow a = 4, \quad \frac{b}{3} = 1 \rightarrow b = 3$ $\frac{b}{a} = \frac{3}{4} = \frac{1}{\frac{4}{3}} \Rightarrow D_f = [\frac{1}{\frac{4}{3}}, 1]$	7
الف) $y = \sqrt{\frac{x^2-x-4}{x^2-13x^2+34}} = \sqrt{\frac{(x^2-4)(x+1)^{-2}}{(x^2-4)(x^2-2)}} \xrightarrow{\pm 2} \frac{-2}{\frac{1}{1}-\frac{1}{1}-\frac{1}{1}+\frac{1}{1}}$ $D_f = (-\infty, -2) \cup (2, 3) \cup (3, +\infty)$ ب) $y = \sqrt{\frac{x^2-2x^2-2x+4}{x^2+2x^2-2x-4}} = \sqrt{\frac{(x-1)(x^2-x-4)}{(x+1)(x^2+x-4)}} = \sqrt{\frac{(x-1)(x^2-4)(x+1)^{-2}}{(x+1)(x+3)(x-2)^2}}$ $\frac{-2}{\frac{1}{1}-\frac{1}{1}+\frac{1}{1}-\frac{1}{1}} + \frac{-1}{\frac{1}{1}-\frac{1}{1}+\frac{1}{1}-\frac{1}{1}} + \frac{2}{\frac{1}{1}-\frac{1}{1}+\frac{1}{1}-\frac{1}{1}} \rightarrow D_f = (-\infty, -2) \cup [-2, -1) \cup (1, 2) \cup (2, +\infty)$	8
الف) $y = \sqrt{2[x]-2} \rightarrow 2[x]-2 \geq 0 \rightarrow 2[x] \geq 2 \rightarrow [x] \geq 1 \Rightarrow D_f = [1, +\infty)$ ب) $y = \sqrt{2[x]-2} \rightarrow 2[x]-2 \geq 0 \rightarrow 2[x] \geq 2 \rightarrow [x] \geq 1 \Rightarrow x \geq 1$ $\Rightarrow D_f = [1, +\infty)$	9
الف) $y = \sqrt{\frac{x+1}{1x+1}} + \sqrt{\frac{4-x}{x^2+2x+2}} \rightarrow \frac{x+1}{1x+1} \geq 0 \rightarrow x \geq -1, x \neq -1, \quad \frac{4-x}{x^2+2x+2} \geq 0 \rightarrow x \leq 4$ $D_f = [-1, 4] - \{-1\}$ ب) $y = \sqrt{[x]-2} + \sqrt{4-[x]} \rightarrow [x]-2 \geq 0 \rightarrow [x] \geq 2 \rightarrow x \geq 2$ $\hookrightarrow 4-[x] \geq 0 \rightarrow [x] \leq 4 \rightarrow x < 5 \Rightarrow D_f = \emptyset$	10