

1) $y = \frac{n+3}{n^3+n^2-n+3} = \frac{n+3}{(n-1)(n-\frac{1}{2})(n+3)}$ $\rightarrow D_f = \mathbb{R} - \{1, \frac{1}{2}, -3\}$

$\downarrow$ $n=1$ $\downarrow$ $n=\frac{1}{2}$ $\downarrow$ $n=-3$

$\frac{n^3+n^2-n+3}{-n^3+n^2} \cdot \frac{(n-1)}{(n-1)} = \frac{n^3+n^2-n+3}{n^3+n^2} \cdot \frac{(n-1)}{(n-1)} \rightarrow n^3+n^2-n+3 = 0 \rightarrow (n-1)(n+3) = 0 \rightarrow n=1$
 $\downarrow$ $n=-3$

$\frac{n^3+n^2-n+3}{-n^3+n^2} = \frac{n^3+n^2-n+3}{-n^3+n^2} \cdot \frac{(n-1)}{(n-1)} = \frac{n^3+n^2-n+3}{-n^3+n^2} \cdot \frac{(n-1)}{(n-1)}$

2) $y = \frac{n+3}{n^3+9n^2+6n+3} = \frac{n+3}{(n+1)(n+3)(n+\frac{1}{2})}$ $\rightarrow D_f = \mathbb{R} - \{-1, -3, -\frac{1}{2}\}$

$\downarrow$ $n=-1$ $\downarrow$ $n=-3$ $\downarrow$ $n=-\frac{1}{2}$

$\frac{n^3+9n^2+6n+3}{-n^3+6n^2} \cdot \frac{(n+1)}{(n+1)} = \frac{n^3+9n^2+6n+3}{-n^3+6n^2} \cdot \frac{(n+1)}{(n+1)} \rightarrow n^3+9n^2+6n+3 = 0 \rightarrow (n+1)(n+3) = 0$
 $\downarrow$ $n=-1$ $\downarrow$ $n=-3$

$\frac{n^3+9n^2+6n+3}{-n^3+6n^2} = \frac{n^3+9n^2+6n+3}{-n^3+6n^2} \cdot \frac{(n+1)}{(n+1)} = \frac{n^3+9n^2+6n+3}{-n^3+6n^2} \cdot \frac{(n+1)}{(n+1)}$

3) $y = \frac{n+3}{n^3-2n^2+2n-1} = \frac{n+3}{(n-1)(n^2-n+1)}$ $\rightarrow D_f = \mathbb{R} - \{1\}$

$\downarrow$ $n=1$ $\downarrow$ $n^2-n+1=0$

$\frac{n^3-2n^2+2n-1}{-n^3+2n^2} \cdot \frac{(n-1)}{(n-1)} = \frac{n^3-2n^2+2n-1}{-n^3+2n^2} \cdot \frac{(n-1)}{(n-1)} \rightarrow n^3-2n^2+2n-1 = 0 \rightarrow \Delta = 1-4 = -3 \rightarrow \text{no real roots}$

$\frac{n^3-2n^2+2n-1}{-n^3+2n^2} = \frac{n^3-2n^2+2n-1}{-n^3+2n^2} \cdot \frac{(n-1)}{(n-1)} = \frac{n^3-2n^2+2n-1}{-n^3+2n^2} \cdot \frac{(n-1)}{(n-1)}$

4) $y = \sqrt{\frac{n+3}{n^3-2n^2+2n-1}} = \sqrt{\frac{n+3}{(n-1)(n^2-n+1)}} \rightarrow D_f = (-\infty, 1] \cup (1, +\infty)$

$\downarrow$ $n=1$ $\downarrow$ $n^2-n+1=0$

$$y = \frac{x}{x^2 - 4|x-1| - x + 4}$$

$n \geq 1 \rightarrow x^2 - 4|x-1| - x + 4 = 0 \rightarrow \Delta = 1 - 4 = -3 < 0$
 $n \geq 1 \rightarrow x^2 - 4|x-1| - x + 4 = x^2 - 4(x-1) - x + 4 = x^2 - 5x + 8 = 0 \rightarrow (x-2)(x-4) = 0$
 $n < 1 \rightarrow x^2 - 4|x-1| - x + 4 = x^2 - 4(1-x) - x + 4 = x^2 + 3x = 0 \rightarrow x(x+3) = 0$
 $n \geq 0, n \leq -3$

$$\rightarrow P_f = \mathbb{R} - \{2, 0, -3\}$$

$$\text{مثال ٢) } y = \sqrt{[x] - x} \rightarrow [x] - x \geq 0 \rightarrow [x] \geq x \rightarrow P_f = (-\infty, 0] \quad (5)$$

$$\text{٣) } y = \frac{x^2 + x}{[x]^2 [x] - x} \rightarrow [x]^2 - x[x] - x \neq 0 \rightarrow [x] = t \rightarrow t^2 - xt - x \neq 0 \rightarrow (t-x)(t+x) \neq 0$$

$t \neq x \rightarrow [x] \neq x \rightarrow x \notin [x, x)$
 $t \neq -x \rightarrow [x] \neq -x \rightarrow x \notin (-x, 0]$

$$P_f = \mathbb{R} - [x, x) \cup (-x, 0]$$

$$P_f = (-\infty, -1) \cup [0, 1) \cup [2, +\infty)$$

$$\text{٤) } y = \sqrt{[x - \frac{1}{x}] + [x + \frac{1}{x}]} = \sqrt{[x - \frac{1}{x}] + [x - \frac{1}{x} + 1]} = \sqrt{[x - \frac{1}{x}] + [x - \frac{1}{x}] + 1} \quad (6)$$

$$= \sqrt{2[x - \frac{1}{x}] + 1} \rightarrow 2[x - \frac{1}{x}] + 1 \geq 0 \rightarrow [x - \frac{1}{x}] \geq -\frac{1}{2} \rightarrow [x - \frac{1}{x}] \geq 0$$

$$\rightarrow x - \frac{1}{x} \geq 0 \rightarrow x \geq \frac{1}{x} \rightarrow P_f = [\frac{1}{x}, +\infty)$$

$$\text{٥) } y = \sqrt{[x - \frac{1}{x}] + [\frac{1}{x} - x]} = \sqrt{[x - \frac{1}{x}] + [-(x - \frac{1}{x})]} = [x - \frac{1}{x}] + [-(x - \frac{1}{x})] \geq 0$$

$$\rightarrow x - \frac{1}{x} \in \mathbb{Z} \rightarrow x \in \mathbb{Z} + \frac{1}{x} \rightarrow P_f = \{x \mid x = k + \frac{1}{x}, k \in \mathbb{Z}\}$$

$$\text{مثال ٣) } y = \frac{6 + \pi + 1}{\tan \pi + 1} \rightarrow n \neq k\pi \quad (7)$$

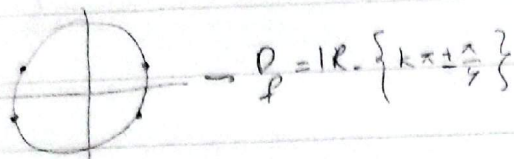
$$n \neq k\pi + \frac{1}{x}$$

$\tan \pi + 1 \neq 0 \rightarrow \tan \pi \neq -1 \rightarrow \pi \neq k\pi + \frac{\pi}{2}$

$$\rightarrow P_f = \mathbb{R} - \left\{ \frac{k\pi}{x}, k\pi + \frac{\pi}{x} \right\}$$

$$\rightarrow y = \sqrt{1 - \varepsilon \sin^2 n} \rightarrow 1 - \varepsilon \sin^2 n \geq 0 \rightarrow (1 - \varepsilon \sin n)(1 + \varepsilon \sin n) \geq 0$$

$$\begin{matrix} \varepsilon \sin n \neq 1 & \varepsilon \sin n \neq -1 \\ \sin n \neq \frac{1}{\varepsilon} & \sin n = -\frac{1}{\varepsilon} \end{matrix}$$



$$\rightarrow D_f = \mathbb{R} \setminus \left\{ k\pi \pm \frac{\pi}{\varepsilon} \right\}$$

$$\rightarrow y = \sqrt{\varepsilon \sin n - 1} \rightarrow \varepsilon \sin n - 1 \geq 0 \rightarrow \sin n \geq \frac{1}{\varepsilon} \rightarrow D_f = \mathbb{R} \setminus \left[k\pi + \frac{\pi}{\varepsilon}, k\pi + \frac{3\pi}{\varepsilon} \right] \quad \textcircled{v}$$



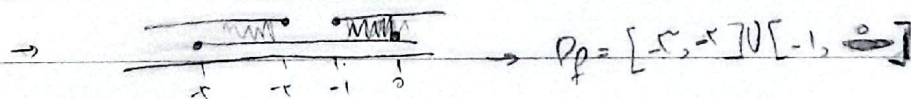
$$\rightarrow y = \sqrt{1 - \varepsilon \cos n} \rightarrow 1 - \varepsilon \cos n \geq 0 \rightarrow \cos n \leq \frac{1}{\varepsilon} \rightarrow D_f = \mathbb{R} \setminus \left[k\pi + \frac{\pi}{\varepsilon}, k\pi + \frac{3\pi}{\varepsilon} \right]$$



$$\rightarrow y = \arcsin(n^2 + \varepsilon n + 1) \rightarrow -1 \leq n^2 + \varepsilon n + 1 \leq 1 \rightarrow n^2 + \varepsilon n + 1 \leq 1 \rightarrow n^2 + \varepsilon n \leq 0 \quad \textcircled{1}$$

$$\rightarrow n(n + \varepsilon) \leq 0 \rightarrow \frac{\varepsilon}{2} \leq n \leq 0$$

$$\rightarrow n^2 + \varepsilon n + 1 \geq -1 \rightarrow n^2 + \varepsilon n + 2 \geq 0 \rightarrow (n + \varepsilon)(n + 1) \geq 0 \rightarrow \frac{\varepsilon}{2} \leq n \leq 0 \rightarrow (-\infty, -\varepsilon] \cup [1, \infty)$$



$$\rightarrow y = \cos y = \sqrt{\varepsilon x n} \rightarrow -1 \leq \sqrt{\varepsilon x n} \leq 1 \rightarrow \sqrt{\varepsilon x n} \leq 1 \rightarrow \varepsilon x n \leq 1 \rightarrow \varepsilon x n \leq 1$$

$$\begin{cases} \varepsilon x n + \varepsilon \leq 0 \rightarrow -n + \varepsilon \leq 0 \rightarrow n \geq \varepsilon \\ \varepsilon x n \geq 0 \rightarrow \varepsilon x n \geq 0 \rightarrow n \leq \frac{\varepsilon}{\varepsilon} \end{cases} \rightarrow D_f = \left[\varepsilon, \frac{\varepsilon}{\varepsilon} \right]$$

$$\rightarrow y = \log_r$$

$$\begin{cases} n > 0 \\ 1 - \log_r n \geq 0 \rightarrow \log_r n < 1 \rightarrow n < r \end{cases} \rightarrow D_f = (0, r) \quad \textcircled{1}$$

$$\rightarrow y = \log_r \left(\frac{1 - \log_r n}{\varepsilon} \right)$$

$$\begin{cases} n > 0 \\ 1 - \log_r \frac{n}{\varepsilon} \geq 0 \rightarrow \log_r \frac{n}{\varepsilon} < 1 \rightarrow n < \frac{\varepsilon}{\varepsilon} \end{cases} \rightarrow D_f = \left(0, \frac{1}{\varepsilon} \right)$$

$$\text{مثال) } y = \sqrt{r^{n+1} - 1} \rightarrow r^{n+1} - 1 \geq 0 \rightarrow r^{n+1} \geq 1 \rightarrow r^{n+1} \geq r \quad (2)$$

$$\rightarrow n^2 - (n+1) \leq 0 \rightarrow (n-1)(n+1) \leq 0 \rightarrow \frac{1}{r-1} \rightarrow P = [1, 2]$$

$$\Rightarrow y = \left(\frac{r^{n+1} - 1}{r-1} \right)! \rightarrow \frac{r^{n+1} - 1}{r-1} \in W \rightarrow r^{n+1} - 1 = (r-1)k \rightarrow r^{n+1} = (r-1)k + 1$$

$$\rightarrow r(r^n - 1) = (r-1)k \rightarrow n = \frac{r^n - 1}{r-1}$$

$$\rightarrow P = \left\{ n \mid n = \frac{r^k - 1}{r-1}, k \in W \right\}$$