

الف) $y = \frac{x+3}{x^2+x^2-2x+3} \Rightarrow x^2+x^2-2x+3 \neq 0 \xrightarrow{\text{مجموعه ضرایب برابر صفر}} (x-1)(2x-1)(x+3)$

$D_f = \mathbb{R} - \{-3, 1/2\}$

$$\begin{array}{r|l} x^2+x^2-2x+3 & 2x-1 \\ \hline -x^2+x^2 & 2x^2+2x-3 \\ \hline -2x+3 & 2x^2+2x-3 \\ \hline -2x+3 & 2x^2+2x-3 \\ \hline 0 & 2x^2+2x-3 \\ \hline 0 & 2x^2+2x-3 \\ \hline 0 & 2x^2+2x-3 \end{array} \Rightarrow (x-1)(2x-1)(x+3)$$

ب) $y = \frac{x+3}{x^2+9x^2+1 \cdot x+3} \Rightarrow x^2+9x^2+1 \cdot x+3 \neq 0 \xrightarrow{\text{مجموعه ضرایب برابر صفر}} (x+1)(2x+1)(x+3)$

$D_f = \mathbb{R} - \{-3, -1, -1/2\}$

$$\begin{array}{r|l} x^2+9x^2+1 \cdot x+3 & x+1 \\ \hline -x^2+9x^2 & 2x^2+2x+3 \\ \hline -x^2+9x^2 & 2x^2+2x+3 \\ \hline -x^2+9x^2 & 2x^2+2x+3 \\ \hline -x^2+9x^2 & 2x^2+2x+3 \\ \hline -x^2+9x^2 & 2x^2+2x+3 \end{array} \Rightarrow (x+1)(2x+1)(x+3)$$

الف) $y = \frac{x+3}{x^2-x^2+2x-1} \Rightarrow x^2-x^2+2x-1 \neq 0 \xrightarrow{\text{مجموعه ضرایب برابر صفر}} (x-1)(x^2-x+1)$

$D_f = \mathbb{R} - \{1\}$

$$\begin{array}{r|l} x^2-x^2+2x-1 & x-1 \\ \hline -x^2+x^2 & x^2-x+1 \\ \hline -x^2+x^2 & x^2-x+1 \\ \hline -x^2+x^2 & x^2-x+1 \\ \hline -x^2+x^2 & x^2-x+1 \\ \hline -x^2+x^2 & x^2-x+1 \end{array} \Rightarrow (x-1)(x^2-x+1)$$

ب) $y = \sqrt{\frac{x+3}{x^2-x^2+2x-1}} \Rightarrow \begin{cases} x+3 \geq 0 \Rightarrow x \geq -3 \\ (x-1)(x^2-x+1) > 0 \Rightarrow x \neq 1 \end{cases}$

$D_f = (-\infty, -3] \cup (1, \infty)$

$y = \frac{x}{x^2-\Delta(x-1)-2x+\Delta} \Rightarrow \begin{cases} x \geq 1 \Rightarrow x^2-\Delta(x-1)-2x+\Delta \geq 0 \Rightarrow x^2-2x-1 \geq 0 \Rightarrow (x-3)(x-1) \\ x \leq -1 \Rightarrow x^2-\Delta(x-1)-2x+\Delta \geq 0 \Rightarrow x^2+3x \geq 0 \Rightarrow x(x+3) \end{cases}$

$D_f = \mathbb{R} - \{-3, 0, 2, \Delta\}$

الف) $y = \sqrt{[x]-2} \Rightarrow [x]-2 \geq 0 \Rightarrow [x] \geq 2 \Rightarrow x \geq 2 \Rightarrow [x] = 2, 3, 4, \dots$

$D_f = [-\infty, -2]$

ب) $y = \frac{\Delta x^2+4}{[x]^2-2[x]-3} \Rightarrow [x]^2-2[x]-3 \neq 0 \Rightarrow ([x]-3)([x]+1) \neq 0$

$D_f = (-\infty, -1) \cup [0, 3) \cup [4, \infty)$

الف) $y = \sqrt{[x-1/4] + [x+1/4]} \Rightarrow [x-1/4] + [x+1/4] \geq 0 \Rightarrow [x-1/4] + [x+1/4] \geq 0 \Rightarrow [x-1/4] + [x+1/4] \geq 0$

$\Rightarrow [x-1/4] \geq -[x+1/4] \Rightarrow x-1/4 \geq -x-1/4 \Rightarrow 2x \geq -1/2 \Rightarrow x \geq -1/4 \Rightarrow [x] \geq 0 \Rightarrow [x] = 0, 1, 2, \dots$

ب) $y = \sqrt{[x-1/4] + [1/4-x]} \Rightarrow \sqrt{[x-1/4] + [1/4-x]} \Rightarrow a \in \mathbb{Z} \Rightarrow [x] + [1/4-x] \geq 0 \Rightarrow x-1/4 \in \mathbb{Z}$

$D_f = \{x | x = k + 1/4, k \in \mathbb{Z}\}$

الف) $y = \frac{\cos x + 1}{\tan x + 1} \quad \begin{cases} \sin x = 0 \rightarrow \{x, 2x\} \\ \cos x = 0 \rightarrow \{\frac{\pi}{2}, \frac{3\pi}{2}\} \\ \tan x = -1 \rightarrow \{\frac{3\pi}{4}, \frac{7\pi}{4}\} \end{cases} \Rightarrow Df = \mathbb{R} - \left\{ \frac{k\pi}{2}, \frac{3\pi}{4} + k\pi \mid k \in \mathbb{Z} \right\}$

ب) $y = \sqrt{1 - 4\sin^2 x} \Rightarrow 1 - 4\sin^2 x \geq 0 \Rightarrow \sin^2 x \leq \frac{1}{4} \Rightarrow -\frac{1}{2} \leq \sin x \leq \frac{1}{2}$

$Df = [k\pi - \frac{\pi}{4}, k\pi + \frac{\pi}{4}]$

الف) $y = \sqrt{r \sin x - 1} \rightarrow r \sin x - 1 \geq 0 \rightarrow \sin x \geq \frac{1}{r} \Rightarrow Df = [rk\pi + \frac{\pi}{4}, rk\pi + \frac{3\pi}{4}]$

ب) $y = \sqrt{1 - 2 \cos x} \rightarrow 1 - 2 \cos x \geq 0 \Rightarrow \cos x \leq \frac{1}{2} \Rightarrow Df = [rk\pi + \frac{\pi}{3}, rk\pi + \frac{5\pi}{3}]$

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الف) $y = \arcsin(x^2 + 2x + 1) \Rightarrow -1 \leq x^2 + 2x + 1 \leq 1 \quad \begin{cases} -1 \leq x^2 + 2x + 1 \Rightarrow x^2 + 2x + 2 \geq 0 \rightarrow (x+1)(x+2) \geq 0 \\ x^2 + 2x + 1 \leq 1 \Rightarrow x^2 + 2x \leq 0 \rightarrow x(x+2) \leq 0 \end{cases}$

$\Rightarrow Df = [-2, -1] \cup [-1, 0]$

ب) $\cos y = \sqrt{a - 2x} \quad \begin{cases} a - 2x \geq 0 \rightarrow x \leq \frac{a}{2} \\ -1 \leq \sqrt{a - 2x} \leq 1 \Rightarrow a - 2x \leq 1 \Rightarrow 2x \geq a - 1 \Rightarrow x \geq \frac{a-1}{2} \end{cases} \xrightarrow{n} [\frac{a-1}{2}, \frac{a}{2}]$

الف) $y = \log_r(1 - \log_r x) \quad \begin{cases} x > 0 \\ 1 - \log_r x > 0 \Rightarrow \log_r x < 1 \Rightarrow x < r \end{cases} \xrightarrow{n} (0, r) = Df$

ب) $y = \log_r(1 - \log_{\frac{1}{r}} x) \quad \begin{cases} x > 0 \\ 1 - \log_{\frac{1}{r}} x > 0 \Rightarrow \log_{\frac{1}{r}} x < 1 \Rightarrow x > \frac{1}{r} \end{cases} \xrightarrow{n} (\frac{1}{r}, \infty) = Df$

الف) $y = \sqrt{r^{x-x^2} - 1} \rightarrow r^{x-x^2} - 1 \geq 0 \rightarrow r^{x-x^2} \geq 1 \rightarrow \frac{r^{x-x^2}}{1} \geq \frac{r^0}{1} \rightarrow r^{x-x^2} \geq r^0 \rightarrow x - x^2 \geq 0 \rightarrow x(1-x) \geq 0 \rightarrow x \in [0, 1]$

$Df = [0, 1]$

ب) $y = \left(\frac{r^x + a}{r^x + r} \right)! \rightarrow \frac{r^x + a}{r^x + r} \in \mathbb{N} \rightarrow r^x + a = r^x + r \Rightarrow x(r, r) = r + r - a$
 $\hookrightarrow x = \frac{-r + a}{r - r} \Rightarrow \frac{-r + a}{r - r} = x$

$Df = \left\{ x \mid x, \frac{r^x + a}{r^x + r}, k \in \mathbb{N} \right\}$