

۲. آخرین خراب نرستی

الف) $2x^3 + 3x^2 - 8x + 3 \neq 0$
 جمع ضرایب $\neq 0$
 بخش بر $(x-1)$

$$\begin{array}{r} 2x^3 + 3x^2 - 8x + 3 \mid x-1 \\ -2x^3 + 2x^2 \\ \hline 5x^2 - 8x + 3 \\ -5x^2 + 5x \\ \hline -3x + 3 \\ +3x - 3 \\ \hline 0 \end{array}$$

$\Delta = 25 - 4 \cdot 5 \cdot 3 = -35$
 $x = \frac{5 \pm \sqrt{-35}}{5} = 1 \pm \frac{\sqrt{-35}}{5}$

$D_f \subset \mathbb{R} - \{1, \frac{1}{5}, \frac{1}{\sqrt{35}}\}$

ب) $2x^3 + 9x^2 + 10x + 3 \neq 0$
 جمع ضرایب توانهای
 خراب جمع ضرایب
 توانهای زوج برابر است
 بخش بر $(x+1)$

$$\begin{array}{r} 2x^3 + 9x^2 + 10x + 3 \mid x+1 \\ -2x^3 - 2x^2 \\ \hline 11x^2 + 10x + 3 \\ -11x^2 - 11x \\ \hline x + 3 \\ -x - 1 \\ \hline 2 \end{array}$$

$\Delta = 100 - 4 \cdot 11 \cdot 3 = -104$
 $x = \frac{-10 \pm \sqrt{-104}}{22} = -\frac{5}{11} \pm \frac{\sqrt{-13}}{11}$

$D_f \subset \mathbb{R} - \{-1, -\frac{1}{11}, -\frac{1}{\sqrt{13}}\}$

الف) $x^3 - 2x^2 + 2x - 1 \neq 0$
 بخش بر $(x-1)$

$$\begin{array}{r} x^3 - 2x^2 + 2x - 1 \mid x-1 \\ -x^3 + x^2 \\ \hline -x^2 + 2x - 1 \\ +x^2 - x \\ \hline x - 1 \\ -x + 1 \\ \hline 0 \end{array}$$

$\Delta < 0 \rightarrow \oplus$ همیشه

$D_f \subset \mathbb{R} - \{1\}$

ب) $x + 3 \neq 0 \rightarrow x \neq -3$
 $x^3 - 2x^2 + 2x - 1 \neq 0 \rightarrow x \neq 1$

$$\begin{array}{r} -3 \quad 1 \\ + \quad - \quad + \quad + \end{array}$$

$D_f \subset \mathbb{R} - \{-3, 1\}$

$x^3 - 5(x-1) - 2x + 5 \geq 0 \rightarrow x^3 - 5x + 5 - 2x + 5 \geq 0$
 $\rightarrow x^3 - 7x + 10 \geq 0 \rightarrow (x-5)(x-2) \geq 0 \rightarrow x \leq 2, x \geq 5$
 $x^3 - 5(-x+1) - 2x + 5 \geq 0 \rightarrow x^3 + 5x - 5 - 2x + 5 \geq 0$
 $\rightarrow x^3 + 3x \geq 0 \rightarrow x(x+3) \geq 0 \rightarrow x \leq -3, x \geq 0$

و $x \geq 1$
 و $x < 1$

$D_f \subset \mathbb{R} - \{-3, 0, 2, 5\}$

الف) $[-x] - 2 \geq 0 \rightarrow [-x] \geq 2 \rightarrow -x \geq 2 \rightarrow x \leq -2 \rightarrow D_f \subset (-\infty, -2]$

ب) $[x]^2 - 2[x] - 3 \neq 0 \rightarrow t^2 - 2t - 3 \neq 0 \rightarrow (t-3)(t+1) \neq 0$
 $\rightarrow [x] \neq -1, [x] \neq 3$
 $D_f \subset \mathbb{R} - \{[3, 4) \cup [-1, 0)\}$

الف) $[x - \frac{1}{x}] + [x + \frac{1}{x}] \geq 0 \rightarrow [x + \frac{1}{x} - 1] + [x + \frac{1}{x}] \geq 0$

$[x + \frac{1}{x}] - 1 + [x + \frac{1}{x}] \geq 0 \rightarrow 2[x + \frac{1}{x}] \geq 1 \rightarrow [x + \frac{1}{x}] \geq \frac{1}{2} \rightarrow x + \frac{1}{x} \geq 1$
 $x \geq \frac{1}{x} \rightarrow D_f \subset [\frac{1}{x}, +\infty)$

ب) $\sqrt{[x - \frac{1}{x}] + [-(x - \frac{1}{x})]} \rightarrow x - \frac{1}{x} \in \mathbb{Z} \rightarrow D_f \subset \{x \mid x = k + \frac{1}{x}, k \in \mathbb{Z}\}$

له آنه نسته صواب ا- می شود

الف) $\tan x + 1 \neq 0 \rightarrow \tan x \neq -1 \rightarrow x \neq k\pi - \frac{\pi}{4}$

ب) $\tan x \rightarrow x \neq k\pi + \frac{\pi}{4}$

الف) $\cot x \rightarrow x \neq k\pi$

$D_f = \mathbb{R} - \{k\pi, k\pi + \frac{\pi}{4}, k\pi - \frac{\pi}{4}\}$

ب) $1 - \sqrt{\sin^2 x} \geq 0 \rightarrow -\sqrt{\sin^2 x} \geq -1 \rightarrow \sqrt{\sin^2 x} \leq 1 \rightarrow \sin^2 x \leq \frac{1}{4} \rightarrow -\frac{1}{2} \leq \sin x \leq \frac{1}{2}$

$D_f = \mathbb{R} - \{k\pi - \frac{\pi}{4}, k\pi + \frac{\pi}{4}\}$



الف) $\sqrt{\sin x} - 1 \geq 0 \rightarrow \sqrt{\sin x} \geq 1 \rightarrow \sin x \geq \frac{1}{4} \rightarrow \sin x \geq \frac{1}{2}$

$D_f = [k\pi + \frac{\pi}{6}, k\pi + \frac{\pi}{2}]$

ب) $1 - \sqrt{\cos x} \geq 0 \rightarrow 1 \geq \sqrt{\cos x} \rightarrow \frac{1}{4} \geq \cos x$



$D_f = [k\pi + \frac{\pi}{3}, k\pi + \frac{2\pi}{3}]$

الف) $-1 \leq x^2 + 3x + 1 \leq 1$

① $-1 \leq x^2 + 3x + 1 \rightarrow 0 \leq x^2 + 3x + 2 \rightarrow 0 \leq (x+2)(x+1) \rightarrow x \in (-\infty, -2] \cup [-1, +\infty)$

② $x^2 + 3x + 1 \leq 1 \rightarrow x^2 + 3x \leq 0 \rightarrow x(x+3) \leq 0 \rightarrow x \in [-3, 0]$

$D_f = [-3, -2] \cup [-1, 0]$

ب) $\Delta - 2x \geq 0 \rightarrow x \leq \frac{\Delta}{2}$

$-1 \leq \Delta - 2x \leq 1 \rightarrow 0 \leq \Delta - 2x \leq 1 \rightarrow \Delta - 1 \leq 2x \leq \Delta \rightarrow \frac{\Delta-1}{2} \leq x \leq \frac{\Delta}{2}$

الف) $1 - \log_{\frac{1}{2}} x > 0 \rightarrow 1 > \log_{\frac{1}{2}} x \rightarrow \frac{1}{2} > x \rightarrow D_f = (0, \frac{1}{2})$

$\hookrightarrow x > 0$

ب) $1 - \log_{\frac{1}{2}} \frac{x}{2} > 0 \rightarrow 1 > \log_{\frac{1}{2}} \frac{x}{2} \rightarrow \frac{1}{2} < \frac{x}{2} \rightarrow D_f = (\frac{1}{2}, +\infty)$

$\hookrightarrow x > 0$

الف) $\sqrt{kx - x^2} - 1 \geq 0 \rightarrow \sqrt{kx - x^2} \geq 1 \rightarrow kx - x^2 \geq 1 \rightarrow kx - x^2 - 1 \geq 0$

$\Delta = 14 - 12 < 4$

$x \in \frac{-k \pm \sqrt{\Delta}}{-2} \in \mathbb{R}, 1$

$\rightarrow D_f = [1, 3]$

ب) $\frac{(kx + \Delta)}{kx + k} \in \mathbb{W} \rightarrow kx + \Delta = kwx + kw \rightarrow kx - kwx = kw - \Delta$

$x(k - kw) = kw - \Delta$

$x = \frac{kw - \Delta}{k - kw}$

$D_f = \{x \mid x = \frac{\Delta - kw}{k - kw}, k \in \mathbb{W}\}$