

الف) $y = \frac{x+3}{2x^3+3x^2-11x+3}$

بجای پذیر بر $x-1$ $\rightarrow$ $(2+3-11+3=0)$ $\rightarrow$ جمع فرایب $= 0$

$2x^3+3x^2-11x+3 \neq 0 \rightarrow (x-1)(2x^2+5x-3) \neq 0$

$\begin{cases} x \neq 1 \\ x \neq -3 \\ x \neq \frac{1}{2} \end{cases}$

$\rightarrow D_f = \mathbb{R} - \{1, -3, \frac{1}{2}\}$

$$\begin{array}{r} 2x^3+3x^2-11x+3 \mid x-1 \\ - 2x^3-2x^2 \\ \hline 5x^2-11x+3 \\ - 5x^2-5x \\ \hline -6x+3 \\ - -6x+3 \\ \hline 0 \end{array}$$

ب) $y = \frac{x+3}{2x^3+9x^2+10x+3}$

بجای پذیر بر $x+1$ $\rightarrow$ $(2+9+10+3=24)$ $\rightarrow$ جمع فرایب توان ها زوج $= 24$

$2x^3+9x^2+10x+3 \neq 0 \rightarrow (x+1)(2x^2+7x+3) \neq 0$

$\begin{cases} x \neq -1 \\ x \neq -3 \\ x \neq -\frac{1}{2} \end{cases}$

$\rightarrow D_f = \mathbb{R} - \{-1, -3, -\frac{1}{2}\}$

$$\begin{array}{r} 2x^3+9x^2+10x+3 \mid x+1 \\ - 2x^3+2x^2 \\ \hline 7x^2+10x+3 \\ - 7x^2+7x+7 \\ \hline 3x+3 \\ - 3x+3 \\ \hline 0 \end{array}$$

الف) $\frac{x+3}{x^3-2x^2+2x-1}$

بجای پذیر بر $x-1$

$x^3-2x^2+2x-1 \neq 0 \rightarrow (x-1)(x^2-x+1) \neq 0 \rightarrow x \neq 1$

$\Delta < 0$ $\rightarrow$ ریشه ندارد

$\rightarrow D_f = \mathbb{R} - \{1\}$

$$\begin{array}{r} x^3-2x^2+2x-1 \mid x-1 \\ - x^3-x^2 \\ \hline -x^2+2x-1 \\ - -x^2+x \\ \hline x-1 \\ - x-1 \\ \hline 0 \end{array}$$

ب) $y = \sqrt{\frac{x+3}{x^3-2x^2+2x-1}}$

$\frac{-3}{+} \mid \frac{1}{-} \rightarrow D_f = (-\infty, -3] \cup (1, +\infty)$

$y = \frac{x}{x^2-5|x-1|-2x+5}$

$x^2-5|x-1|-2x+5 \neq 0$

$\begin{cases} x^2-5x+5-2x+5 \neq 0 \rightarrow x^2-7x+10 \neq 0 \rightarrow (x-2)(x-5) \neq 0 \rightarrow \begin{cases} x \neq 2 \\ x \neq 5 \end{cases} \\ x^2+5x-5-2x+5 \neq 0 \rightarrow x^2+3x \neq 0 \rightarrow x(x+3) \neq 0 \rightarrow \begin{cases} x \neq 0 \\ x \neq -3 \end{cases} \end{cases}$

$$\text{الف) } y = \sqrt{[-x] - 2} \quad [-x] - 2 \geq 0 \rightarrow [-x] \geq 2 \rightarrow x \leq -2$$

$$\rightarrow D_f = (-\infty, -2]$$

$$\text{ب) } y = \frac{5x^2 + 4}{[x]^2 - 2[x] - 3}$$

$$[x]^2 - 2[x] - 3 \neq 0 \rightarrow ([x] - 3)([x] + 1) \neq 0$$

$$\rightarrow [x] \neq -1 \rightarrow -1 \leq x < 0$$

$$\rightarrow [x] \neq 3 \rightarrow 3 \leq x < 4$$

$$\rightarrow D_f = (-\infty, -1) \cup [0, 3) \cup [4, +\infty)$$

$$\text{الف) } y = \sqrt{[x - \frac{1}{y}] + [x + \frac{y}{y}]}$$

$$[x - \frac{1}{y}] + [x + \frac{y}{y}] \geq 0 \rightarrow [x + \frac{y}{y} - 1] + [x + \frac{y}{y}] \geq 0$$

$$\rightarrow [x + \frac{y}{y}] \geq \frac{1}{y} \rightarrow x + \frac{y}{y} \geq 1 \rightarrow x \geq \frac{1}{y}$$

$$\rightarrow D_f = [\frac{1}{y}, +\infty)$$

$$\text{ب) } y = \sqrt{[x - \frac{1}{y}] + [\frac{1}{y} - x]} = \sqrt{[x - \frac{1}{y}] + [-(x - \frac{1}{y})]} \xrightarrow{\text{if}} \begin{cases} a \in \mathbb{Z} & [a] + [-a] = 0 \\ a \in \mathbb{Z} & [a] + [-a] = -1 \end{cases} \rightarrow x - \frac{1}{y} \in \mathbb{Z}$$

$$\rightarrow D_f = \{x \mid x = k + \frac{1}{y}, k \in \mathbb{Z}\}$$

$$\text{الف) } y = \frac{\cot x + 1}{\tan x + 1}$$

$$\begin{cases} \tan x \neq -1 \\ \cos x \neq 0 \end{cases} \quad \text{Unit Circle Diagram} \rightarrow D_f = \mathbb{R} - \left\{k\pi + \frac{\pi}{4}, k\pi + \frac{3\pi}{4}\right\}$$

$$\text{ب) } y = \sqrt{1 - \varepsilon \sin^2 x}$$

$$1 - \varepsilon \sin^2 x \geq 0 \rightarrow \sin^2 x \leq \frac{1}{\varepsilon} \rightarrow \sin x \leq \frac{1}{\sqrt{\varepsilon}}$$



$$\rightarrow D_f = [2k\pi + \frac{5\pi}{4}, 2k\pi + \frac{7\pi}{4}]$$

$$\text{الف) } y = \sqrt{2 \sin x - 1}$$

$$2 \sin x - 1 \geq 0 \rightarrow \sin x \geq \frac{1}{2}$$



$$\rightarrow D_f = [2k\pi + \frac{\pi}{6}, 2k\pi + \frac{5\pi}{6}]$$

$$\text{ب) } y = \sqrt{1 - 2 \cos x}$$

$$1 - 2 \cos x \geq 0 \rightarrow \cos x \leq \frac{1}{2}$$



$$\rightarrow D_f = [2k\pi + \frac{2\pi}{3}, 2k\pi + \frac{4\pi}{3}]$$

الف) $y = \arcsin(x^2 + 3x + 1)$

$$-1 \leq x^2 + 3x + 1 \leq 1 \Rightarrow \begin{cases} x^2 + 3x + 1 \geq -1 \rightarrow x^2 + 3x + 2 \geq 0 \rightarrow (x+2)(x+1) \geq 0 \\ x^2 + 3x + 1 \leq 1 \rightarrow x^2 + 3x \leq 0 \rightarrow x(x+3) \leq 0 \end{cases}$$

$$\rightarrow D_f = [-2, -1] \cup [-1, 0]$$

ب) $\cos y = \sqrt{5-2x}$

$$\begin{cases} 5-2x \geq 0 \rightarrow x \leq \frac{5}{2} \\ -1 \leq \sqrt{5-2x} \leq 1 \rightarrow 5-2x \leq 1 \rightarrow x \geq 2 \end{cases} \Rightarrow D_f = [2, \frac{5}{2}]$$

الف) $y = \log_x(1 - \log_x x)$

$$\begin{cases} 1 - \log_x x > 0 \rightarrow \log_x x < 1 \rightarrow x < x \\ x > 0 \end{cases} \Rightarrow D_f = (0, x)$$

ب) $y = \log_x(1 - \log_x \frac{x}{x})$

$$\begin{cases} 1 - \log_x \frac{x}{x} > 0 \rightarrow \log_x \frac{x}{x} < 1 \rightarrow x > \frac{1}{x} \\ x > 0 \end{cases} \Rightarrow D_f = (\frac{1}{x}, +\infty)$$

الف) $y = \sqrt{x^{\epsilon x - x^x} - 1}$

$$\begin{aligned} x^{\epsilon x - x^x} - x^x &\geq 0 \rightarrow x^x (x^{\epsilon x - x^x - x} - 1) \geq 0 \\ \rightarrow x^{-x^x + \epsilon x - x} &\geq 1 \rightarrow -x^x + \epsilon x - x \geq 0 \rightarrow (x-3)(x-1) \geq 0 \end{aligned}$$

$$\frac{1}{-} \frac{3}{+} \frac{1}{-} \rightarrow D_f = [1, 3]$$

ب) $y = \left(\frac{x^x + 5}{x^x + \epsilon} \right)!!$

$$\frac{x^x + 5}{x^x + \epsilon} \in W \rightarrow x = -\frac{\epsilon W - 5}{x^x - \epsilon} = \frac{-\epsilon W + 5}{x^x - \epsilon}$$

$$\rightarrow D_f = \left\{ x \mid x = \frac{-\epsilon k + 5}{x^x - \epsilon}, k \in W \right\}$$