

۲۰. از بین عبارتی دراصل نوشته شده

ب) $f(x) = \frac{1}{x+1} - \frac{1}{x}$ $\rightarrow x+1 \neq 0 \rightarrow x \neq -1$
 $\frac{1}{x-1} + \frac{1}{x+1}$ $\rightarrow x-1 \neq 0 \rightarrow x \neq 1$
 $\frac{1}{x-1} + \frac{1}{x+1} = \frac{1 \cdot x + 1 \cdot x - 1}{(x-1)(x+1)} = \frac{x+1}{(x-1)(x+1)}$
 $\rightarrow x+1 \neq 0 \rightarrow x \neq -1$
 $Df = \mathbb{R} - \{-1, -\frac{1}{2}, -1, 0, 1\}$

الف) $f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}}$
 $\rightarrow \frac{x-1}{x} - \frac{x}{x-1} \geq 0 \rightarrow \frac{x^2 - 1 - x^2}{x(x-1)} \geq 0$
 $\rightarrow \frac{1 - 2x + \frac{1}{x}}{x(x-1)} \geq 0$
 $\rightarrow Df = (-\infty, 0) \cup [\frac{1}{2}, 1)$

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ب) $\sqrt{x-1} + \sqrt{y+1} = 1$
 $\rightarrow x-1 \geq 0 \rightarrow x \geq 1$
 $\rightarrow \sqrt{y+1} = 1 - \sqrt{x-1} \rightarrow 1 - \sqrt{x-1} \geq 0$
 $\rightarrow \sqrt{x-1} \leq 1 \rightarrow x-1 \leq 1 \rightarrow x \leq 2$
 $Df = [1, 2]$

الف) $f(x) = \sqrt{((\frac{1}{10})^x - 9)(10^x - 4)}$
 $\rightarrow (\frac{1}{10} - 9)(10 - 4) \geq 0$
 $\rightarrow Df = (-\infty, -2] \cup [\frac{1}{10}, +\infty)$

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$f(x) = \frac{\log(x^2 - x - 2)}{\sqrt{x^2 - 1} + 1}$ $\rightarrow x^2 - x - 2 > 0 \rightarrow (x-2)(x+1) > 0$
 $\rightarrow \sqrt{x^2 - 1} + 1 \neq 0 \rightarrow \sqrt{x^2 - 1} \neq -1$
 $\rightarrow x^2 - 1 \geq 0 \rightarrow (x+1)(x-1) \geq 0$
 $Df = (-\infty, -1) \cup (2, +\infty)$

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$f(x) = \sqrt{13 + ax - x^2}$ $\rightarrow Df = [-2, b]$
 $x = -2 \rightarrow 13 + a(-2) - (-2)^2 = 0 \rightarrow 13 - 2a - 4 = 0 \rightarrow -2a - 1 = 0 \rightarrow 2a = -1 \rightarrow a = -\frac{1}{2}$
 $\rightarrow -x^2 - \frac{1}{2}x + 13 = 0 \rightarrow \Delta = b^2 - 4ac = \frac{1}{4} - 4(-1)(13) = \frac{1}{4} + \frac{52}{1} = \frac{209}{4}$
 $\rightarrow \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{\frac{1}{2} \pm \frac{\sqrt{209}}{2}}{-1} \rightarrow \frac{1 \pm \sqrt{209}}{-2} = -2 \rightarrow b = \frac{1}{2}$
 $a + b = -\frac{1}{2} + \frac{1}{2} = 0$

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$f(x) = \begin{cases} 13x - 2 & x \geq 1 \\ 13x + 13 & x < 1 \end{cases} \rightarrow g(x) = \sqrt{f(x) - x}$
 $\rightarrow f(x) - x \geq 0 \rightarrow \begin{cases} 12x - 2 & x \geq 1 \\ 12x + 13 & x < 1 \end{cases}$
 $Dg = [-13, +\infty)$

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$$f(x) = x^2 - 4x$$

$$f\left(\sin \frac{\pi}{\sqrt{2}} + \cos \frac{\pi}{\sqrt{2}}\right) = f\left(\frac{\sqrt{2}}{\sqrt{2}} + 1\right) = 2\left(\frac{\sqrt{2}}{\sqrt{2}} + 1\right)^2 - 4\left(\frac{\sqrt{2}}{\sqrt{2}} + 1\right) =$$

$$\rightarrow 2\left(\frac{2}{2} + 1 + \sqrt{2}\right) - 4\sqrt{2} - 4 = \frac{2}{2} + 2 + 2\sqrt{2} - 4\sqrt{2} - 4 = 1 - 2 = -1$$

$$\rightarrow \boxed{-\frac{1}{2}}$$

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$$f(x) = \frac{x+1}{x-1} \rightarrow \text{به ازای } x=a \text{ مقدارگیری از } a$$

$$f(f(a)) = a$$

$$\rightarrow f(a) = \frac{a+1}{a-1} \rightarrow f\left(\frac{a+1}{a-1}\right) = \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = \frac{\frac{a+1+a-1}{a-1}}{\frac{a+1-a+1}{a-1}} = \frac{2a}{2} = a$$

$$\rightarrow \boxed{\mathbb{R} - \{1\}}$$

! به ازای هم مقدارگیری به چیز! زیرا به ازای! تعریف نشده است!

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$$f(x) = x^2 - 4x$$

$$g(x) = \sqrt{x^2 - 1}$$

$$f(g(a)) = -4$$

$$f(x) = -4 \rightarrow x^2 - 4x = -4 \rightarrow x^2 - 4x + 4 = 0$$

$$\rightarrow (x-2)^2 = 0 \rightarrow x = 2$$

$$\rightarrow g(a) = 2 \rightarrow \sqrt{a^2 - 1} = 2 \rightarrow \sqrt{a^2 - 1} = \sqrt{4} \rightarrow a^2 - 1 = 4$$

$$\rightarrow a^2 = 5 \rightarrow \boxed{a = \pm \sqrt{5}}$$

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$$f(x+y) = x^2 - 2xy + y^2$$

$$f(1) \times f(-3) = ?$$

$$\rightarrow f(1) = f(1+3) = 1^2 - 2 \times 1 \times 3 + 3^2 = 1 - 6 + 9 = 4$$

$$\rightarrow f(-3) = f(-4+3) = (-4)^2 - 2(-4) \times 3 + 3^2 = 16 + 24 + 9 = 49$$

$$\rightarrow f(1) \times f(-3) = 4 \times 49 = \boxed{196}$$

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$$f(\cos^2 x + \sin^2 x) = \tan^2 x$$

$$f(3, 5) = ?$$

$$\cos x = \frac{1}{\sqrt{2}} \rightarrow f\left(\cos^2 x + \sin^2 x\right) = f\left(1 + \frac{1}{2}\right) = f\left(\frac{3}{2}\right) = \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}$$

$$\rightarrow f(3, 5) = \boxed{\frac{1}{2}}$$

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