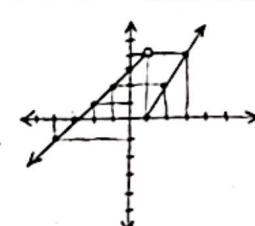


|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |                                                                                                                                                                                                                                                                                                                                                           |          |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------|
| <p>ب) <math>f(x) = \frac{1}{x+1} - \frac{1}{x}</math> <math>\sim x+1 \neq 0 \sim x \neq -1</math><br/> <math>\frac{1}{x-1} + \frac{1}{x+1}</math> <math>\sim x-1 \neq 0 \sim x \neq 1</math><br/> <math>\frac{1}{x-1} + \frac{1}{x+1} = \frac{1(x+1) + 1(x-1)}{(x-1)(x+1)} = \frac{x+1+x-1}{(x-1)(x+1)} = \frac{2x}{(x-1)(x+1)}</math><br/> <math>\sim 2x \neq 0 \sim x \neq 0</math><br/> <math>Df = \mathbb{R} - \{-1, -\frac{1}{2}, 0, 1\}</math></p>                                         | <p>الف) <math>f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}}</math><br/> <math>\sim \frac{x-1}{x} - \frac{x}{x-1} \geq 0 \sim \frac{x^2 - 1 - x^2}{x(x-1)} \geq 0</math><br/> <math>\sim \frac{-1}{x(x-1)} \geq 0 \sim \frac{-1}{x(x-1)} \geq 0</math><br/> <math>\sim Df = (-\infty, 0) \cup [\frac{1}{2}, 1)</math></p>                                    | <p>۱</p> |
| <p>ب) <math>\sqrt{x-1} + \sqrt{y+1} = 1</math><br/> <math>\sim x-1 \geq 0 \sim x \geq 1</math><br/> <math>\sim \sqrt{y+1} = 1 - \sqrt{x-1} \sim 1 - \sqrt{x-1} \geq 0</math><br/> <math>\sim \sqrt{x-1} \leq 1 \sim x-1 \leq 1 \sim x \leq 2</math><br/> <math>\sim Df = [1, 2]</math></p>                                                                                                                                                                                                       | <p>الف) <math>f(x) = \sqrt{((\frac{1}{10})^x - 9)(10^x - 4)}</math><br/> <math>\sim (\frac{1}{10})^x - 9 \geq 0 \sim (\frac{1}{10})^x \geq 9 \sim x \leq \log_{10} \frac{1}{9}</math><br/> <math>\sim 10^x - 4 \geq 0 \sim 10^x \geq 4 \sim x \geq \log_{10} 4</math><br/> <math>\sim Df = (-\infty, -\log_{10} 9] \cup [\log_{10} 4, +\infty)</math></p> | <p>۲</p> |
| <p><math>f(x) = \frac{\log(x^2 - x - 2)}{\sqrt{x^2 - 1} + 1}</math> <math>\sim x^2 - x - 2 &gt; 0 \sim (x-2)(x+1) &gt; 0</math><br/> <math>\sim \sqrt{x^2 - 1} + 1 \neq 0 \sim \sqrt{x^2 - 1} \neq -1</math><br/> <math>\sim x^2 - 1 \geq 0 \sim (x+1)(x-1) \geq 0</math><br/> <math>\sim Df = (-\infty, -1) \cup (2, +\infty)</math></p>                                                                                                                                                        | <p>همواره برقرار است!<br/> <math>\sim Df = (-\infty, -1) \cup (2, +\infty)</math></p>                                                                                                                                                                                                                                                                     | <p>۳</p> |
| <p><math>f(x) = \sqrt{1 + ax - x^2} \sim Df = [-1, b]</math><br/> <math>x = -1 \sim 1 + a(-1) - (-1)^2 = 0 \sim 1 - a - 1 = 0 \sim -a = 0 \sim a = 0</math><br/> <math>\sim -x^2 - \frac{1}{2}x + 1 = 0 \sim \Delta = b^2 - 4ac = \frac{1}{4} - 4(-1)(1) = \frac{1}{4} + 4 = \frac{17}{4}</math><br/> <math>\sim \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{\frac{1}{2} \pm \frac{\sqrt{17}}{2}}{-2} \sim \frac{1 \pm \sqrt{17}}{-4}</math><br/> <math>\sim b = \frac{1 + \sqrt{17}}{-4}</math></p> | <p><math>a + b = \frac{1}{2} - \frac{1}{2} = 0</math></p>                                                                                                                                                                                                                                                                                                 | <p>۴</p> |
| <p><math>f(x) = \begin{cases} 1^x - 1 &amp; x \geq 1 \\ 1^x + 1 &amp; x &lt; 1 \end{cases} \sim g(x) = \sqrt{f(x) - x}</math><br/> <math>\sim f(x) - x \geq 0 \sim \begin{cases} 1^x - 1 - x &amp; x \geq 1 \\ 1^x + 1 - x &amp; x &lt; 1 \end{cases}</math><br/> <math>Dg = [-1, +\infty)</math></p>                                                                                                                                                                                            |                                                                                                                                                                                                                                                                       | <p>۵</p> |

$$f(x) = x^2 - 4x$$

$$f\left(\sin \frac{\pi}{\sqrt{2}} + \cos \sqrt{2}\pi\right) = f\left(\frac{\sqrt{2}}{\sqrt{2}} + 1\right) = 2\left(\frac{\sqrt{2}}{\sqrt{2}} + 1\right)^2 - 4\left(\frac{\sqrt{2}}{\sqrt{2}} + 1\right) =$$

$$\rightarrow 2\left(\frac{2}{2} + 1 + \sqrt{2}\right) - 2\sqrt{2} - 4 = \frac{2}{2} + 2 + 2\sqrt{2} - 2\sqrt{2} - 4 = 1 - 2 = -1$$

$$\rightarrow \boxed{-\frac{1}{2} = -0.5}$$

6

$$f(x) = \frac{x+1}{x-1} \rightarrow \text{به ازای } x \text{ مقدارگیری از } a$$

$$f(f(a)) = a$$

$$\rightarrow f(a) = \frac{a+1}{a-1} \rightarrow f\left(\frac{a+1}{a-1}\right) = \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = \frac{\frac{a+1+a-1}{a-1}}{\frac{a+1-a+1}{a-1}} = \frac{2a}{2} = a$$

$$\rightarrow \boxed{\mathbb{R} - \{1\}}$$

! به ازای هم مقدارگیری به چیز! / زیرا به ازای! تعریف نشده است!

7

$$f(x) = x^2 - 4x$$

$$g(x) = \sqrt{x^2 - 1}$$

$$f(g(a)) = -4$$

$$f(x) = -4 \rightarrow x^2 - 4x = -4 \rightarrow x^2 - 4x + 4 = 0$$

$$\rightarrow (x-2)^2 = 0 \rightarrow x = 2$$

$$\rightarrow g(a) = 2 \rightarrow \sqrt{a^2 - 1} = 2 \rightarrow \sqrt{a^2 - 1} = \sqrt{4} \rightarrow a^2 - 1 = 4$$

$$\rightarrow a^2 = 5 \rightarrow \boxed{a = \pm \sqrt{5}}$$

8

$$f(x+y) = x^2 - 2xy + y^2$$

$$f(1) \times f(-3) = ?$$

$$\rightarrow f(1) = f(1+3) = 1^2 - 2 \times 1 \times 3 + 3^2 = 1 - 6 + 9 = 4$$

$$\rightarrow f(-3) = f(-4+3) = (-4)^2 - 2(-4) \times 3 + 3^2 = 16 + 24 + 9 = 49$$

$$\rightarrow f(1) \times f(-3) = 4 \times 49 = \boxed{196}$$

9

$$f\left(\frac{1}{2} \cos^2 x + \frac{1}{2} \cos x\right) = \tan^2 x$$

$$f\left(\frac{1}{2}, \frac{1}{2}\right) = ?$$

$$\cos x = \frac{1}{2} \rightarrow f\left(\frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2}\right) = f\left(\frac{1}{2} + \frac{1}{2}, \frac{1}{2}\right) = f\left(\frac{1}{2}, \frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$$\rightarrow f\left(\frac{1}{2}, \frac{1}{2}\right) = \boxed{\frac{1}{4}}$$

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