

۲. آزمون یکی نفر

الف)  $f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}}$

$D_f = (-\infty, 0) \cup (\frac{1}{2}, 1)$

ب)  $f(x) = \frac{\frac{1}{x+1} - \frac{x}{x}}{\frac{x}{x-1} + \frac{1}{x+2}}$

$D_f = \mathbb{R} - \{-1, -\frac{1}{2}, \frac{1}{2}, 1\}$

الف)  $f(x) = \sqrt{(\frac{1}{x}-9)(x^2-x^2)}$

$D_f = (-\infty, -2] \cup [\frac{1}{2}, \infty)$

ب)  $\sqrt{x-1} + \sqrt{y+1} = 3$

$f(x) = \frac{\log(x^2-x-2)}{\sqrt{x^2-1} + 1}$

$D_f = (-\infty, -1) \cup (2, \infty)$

$f(x) = \sqrt{x+ax-x^2}$   $D_f = [-2, b]$   $a+b = ?$

$-x^2+ax+3 \geq 0$   $f(-2) \geq 0 \Rightarrow -4-2a+3 \geq 0 \Rightarrow -2a \geq 1 \Rightarrow a \leq -\frac{1}{2}$

$-x^2 - \frac{1}{2}x + 3 \geq 0$   $x \geq \frac{\frac{1}{2} \pm \sqrt{\frac{1}{4} + 12}}{-2} = \frac{-1 \pm \sqrt{49}}{-2} = \frac{-1 \pm 7}{-2}$

$a+b \leq \frac{3}{2} + (-\frac{1}{2}) = 1$

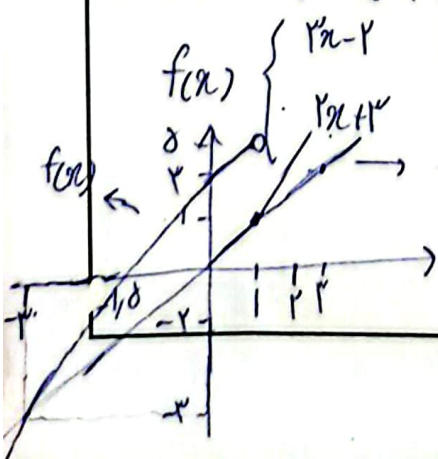
$f(x) = \begin{cases} 2x-2 & x \geq 1 \\ 2x+3 & x < 1 \end{cases}$

$g(x) = \sqrt{f(x)-x}$

$f(x)-x \geq 0 \Rightarrow f(x) \geq x$

$2x-2 \geq x \Rightarrow x \geq 2$   $2x+3 \geq x \Rightarrow x \geq -3$

$D(g) = [-3, \infty)$



$$f(x) = x^r - rx$$

$$f\left(\sin \frac{\pi}{p} + \cos r\pi\right) = f\left(\frac{\sqrt{p}}{p} + 1\right) = f\left(\frac{1+\sqrt{p}}{p}\right)$$

$$f\left(\frac{1+\sqrt{p}}{p}\right) = \left(\frac{1+\sqrt{p}}{p}\right)^r - r\left(\frac{1+\sqrt{p}}{p}\right) = \left(\frac{1+\sqrt{p}}{p^r}\right) - r - \frac{r\sqrt{p}}{p} = \frac{1}{p} + \frac{r\sqrt{p}}{p} - r - \frac{r\sqrt{p}}{p} = \frac{1-r}{p}$$

$$f(x) = \frac{x+1}{x-1} \quad f(f(a)) = a \quad f\left(\frac{a+1}{a-1}\right) = a$$

$$\frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = \frac{\frac{2a}{a-1}}{\frac{2}{a-1}} = a \quad a \neq 1 \quad (a-1 \neq 0) \quad \frac{2}{a-1} \neq 0 \quad \boxed{a \in \mathbb{R} - \{1\}}$$

$$f(x) = x^r - rx \quad f(g(a)) = -r$$

$$g(x) = \sqrt{x^r - 1}$$

$$x^r - rx = -r$$

$$x^r - rx + r = 0 \quad (x-r)^r = 0$$

$$x = r \Rightarrow g(a) = r$$

$$\sqrt{a^r - 1} = r \quad a^r - 1 = r^2 \quad a^r = r^2 + 1 \quad \boxed{a = \pm \sqrt[r]{r^2 + 1}}$$

$$f(x+y) = x^r - dx + y = (x-y)(x+y)$$

$$a = x+y \quad x = a-y \quad f(a) = (a-y)(a-d)$$

$$f(r) \times f(-r) = r \times (-r) = -r^2$$

$$f(r) = (-r) \times (-1) = r$$

$$f(-r) = (-r) \times (-1) = r$$

$$f(r \cos \theta + d \cos \theta) = \tan^r \theta$$

$$f\left(\frac{r}{p}\right) = \tan^r\left(\frac{\pi}{p}\right) = \left(\frac{r}{p}\right)^r = \frac{r^r}{p^r}$$

$$r \cos^r \theta + d \cos \theta = \frac{r}{p}$$

$$\cos \theta = \frac{-d \pm \sqrt{d^2 + \frac{r^2}{p^2}}}{r}$$

$$\frac{1}{r} \rightarrow \frac{1}{p} \rightarrow \frac{1}{r} \times \frac{r}{p} = \frac{1}{p}$$

$$x = \frac{\pi}{p}$$

$$-1 \leq \cos \theta \leq 1$$