

$$الف) f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}}$$

$$D_f = (-\infty, 0) \cup (\frac{1}{x}, 1)$$

$$\frac{x-1}{x} - \frac{x}{x-1} \geq 0 \Rightarrow \frac{(x-1)^2 - x^2}{x(x-1)} \geq 0 \Rightarrow \frac{1-x^2}{x(x-1)} \geq 0$$

$$ب) f(x) = \frac{\frac{1}{x+1} - \frac{x}{x}}{\frac{x}{x-1} + \frac{1}{x+1}} \Rightarrow$$

$$\frac{x}{x-1} + \frac{1}{x+1} \neq 0 \Rightarrow \frac{x^2 + x + x + 1}{(x-1)(x+1)} \neq 0$$

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$$D_f = R - \{-1, \frac{1}{x}\} = (-\infty, -1) \cup (-1, 0) \cup (0, 1) \cup (1, \infty)$$

$$x \neq -1 (x+1 \neq 0)$$

$$x \neq \frac{1}{x} (x^2 \neq 1)$$

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$$الف) f(x) = \sqrt{(\frac{1}{x} - 9)(x^2 - x^2)}$$

$$((\frac{1}{x} - 9)(x^2 - x^2)) \geq 0 \Rightarrow \frac{-x}{x} \geq 0 \Rightarrow -1 \geq 0$$

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$$D_f = (-\infty, -1] \cup [\frac{1}{9}, \infty)$$

$$ب) \sqrt{x-1} + \sqrt{y+1} = 3$$

$$\sqrt{x-1} \geq 0 \Rightarrow x \geq 1$$

$$\sqrt{y+1} \geq 0 \Rightarrow y \geq -1$$

$$\sqrt{x-1} + \sqrt{y+1} = 3 \Rightarrow \sqrt{x-1} = 3 - \sqrt{y+1}$$

$$f(x) = \frac{\log(x^2 - x - 2)}{\sqrt{x^2 - 1} + 1}$$

$$x^2 - x - 2 > 0 \Rightarrow x < -1 \text{ or } x > 2$$

$$\sqrt{x^2 - 1} + 1 \neq 0 \Rightarrow \sqrt{x^2 - 1} \neq -1$$

$$(x+1)(x-2) > 0 \Rightarrow x < -1 \text{ or } x > 2$$

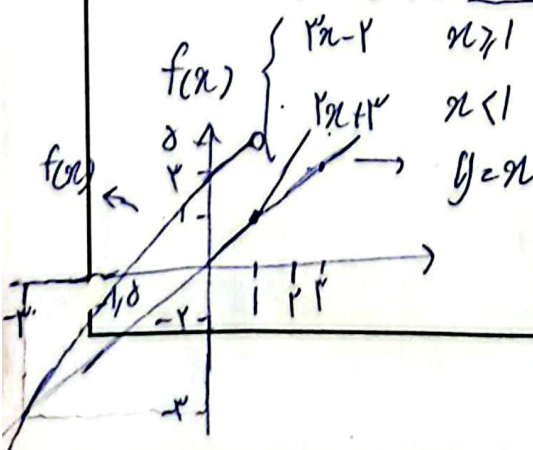
$$D_f = (-\infty, -1) \cup (2, \infty)$$

$$f(x) = \sqrt{x + ax - x^2} \quad D_f = [-1, b] \quad a+b = ?$$

$$-x^2 + ax + x \geq 0 \Rightarrow f(-1) \geq 0 \Rightarrow -1 - 1a + 1 \geq 0 \Rightarrow -1a \geq 0 \Rightarrow a \leq 0$$

$$-x^2 + \frac{1}{x}x + x \geq 0 \Rightarrow x \geq \frac{1}{x} \Rightarrow x^2 \geq 1 \Rightarrow x \geq 1 \text{ or } x \leq -1$$

$$a+b = \frac{1}{x} + (-\frac{1}{x}) = 0$$



$$g(x) = \sqrt{f(x) - x}$$

$$f(x) - x \geq 0 \Rightarrow f(x) \geq x$$

$$x^2 - x \geq x \Rightarrow x^2 - 2x \geq 0 \Rightarrow x(x-2) \geq 0 \Rightarrow x \leq 0 \text{ or } x \geq 2$$

$$D(g) = [-1, \infty)$$

$$f(x) = x^r - rx$$

$$f\left(\sin \frac{\pi}{p} + \cos r\pi\right) = f\left(\frac{\sqrt{p}}{p} + 1\right) = f\left(\frac{1+\sqrt{p}}{p}\right)$$

$$f\left(\frac{1+\sqrt{p}}{p}\right) = \left(\frac{1+\sqrt{p}}{p}\right)^r - r\left(\frac{1+\sqrt{p}}{p}\right) = \left(\frac{1+\sqrt{p}}{p}\right)^r - r - r\sqrt{p} = \frac{1}{p} + \frac{\sqrt{p}}{p} - r - r\sqrt{p} = \frac{1}{p} - r - r\sqrt{p} + \frac{\sqrt{p}}{p}$$

$$f(x) = \frac{x+1}{x-1} \quad f(f(a)) = a \quad f\left(\frac{a+1}{a-1}\right) = a$$

$$\frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = \frac{\frac{2a}{a-1}}{\frac{2}{a-1}} = a \quad a \neq 1 \quad (a-1 \neq 0) \quad \boxed{a \in \mathbb{R} - \{1\}}$$

$$f(x) = x^r - rx \quad f(g(a)) = -r$$

$$g(x) = \sqrt{x^r - 1}$$

$$x^r - rx = -r$$

$$x^r - rx + r = 0 \quad (x-r)^r = 0$$

$$x = r \Rightarrow g(a) = r$$

$$\sqrt{a^r - 1} = r \quad a^r - 1 = r^2 \quad a^r = r^2 + 1 \quad \boxed{a = \pm \sqrt[r]{r^2 + 1}}$$

$$f(x+y) = x^r - dx + y = (x-y)(x+y)$$

$$a = x+y \quad x = a-y \quad f(a) = (a-y)(a-d)$$

$$f(r) \times f(-r) = r \times (-r) = -r^2$$

$$f(r) = (-r) \times (-1) = r$$

$$f(-r) = (-r) \times (-1) = r$$

$$f(r \cos \theta + d \cos \theta) = \tan \theta$$

$$f\left(\frac{r}{p}\right) = \tan\left(\frac{\pi}{p}\right) = \left(\frac{r}{p}\right)^r = \frac{r^r}{p^r}$$

$$r \cos \theta + d \cos \theta = \frac{r}{p}$$

$$\cos \theta = \frac{-d \pm \sqrt{d^2 + r^2}}{r}$$

$$\frac{1}{r} \rightarrow \frac{1}{p}$$

$$x = \frac{\pi}{p}$$

$$-1 \leq \cos \theta \leq 1$$