

الف) $f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}} \rightarrow \sqrt{\frac{(x-1)^2 - x^2}{x(x-1)}} \rightarrow \sqrt{\frac{-x^2+1}{x(x-1)}} \rightarrow \sqrt{\frac{-x^2+1}{x^2-x}}$

$D_f = (-\infty, 0) \cup [\frac{1}{x}, 1)$

ب) $f(x) = \frac{\frac{1}{x+1} - \frac{x}{x}}{\frac{x}{x-1} + \frac{1}{x+2}} \Rightarrow \frac{\frac{1-x}{x(x+1)}}{\frac{x^2+x+1}{(x-1)(x+2)}} = \frac{(1-x)(x+2)}{x(x+1)(x^2+x+1)}$

$D_f = (-2, -1) \cup (0, 1) \cup (1, \infty)$

الف) $f(x) = \sqrt{\left(\left(\frac{1}{x}\right)^x - 1\right)\left(x^2 - 1\right)} \sim \left(\frac{1}{x}\right)^x \geq 0 \rightarrow x^x = \left(\frac{1}{x}\right)^{-x} \rightarrow x \geq -2$

$x^2 \geq 1 \rightarrow x \geq 1 \text{ or } x \leq -1 \rightarrow x \leq \frac{\infty}{x} \rightarrow D_f = [-2, \infty]$

ب) $\sqrt{x-1} + \sqrt{y+1} = 3 \rightarrow x-1 \geq 0 \rightarrow x \geq 1$ $D_f = [1, \infty)$

$y+1 \geq 0 \rightarrow y \geq -1$

$f(x) = \frac{\log_e(x^2-x-2)}{\sqrt{x^2-1} + 1} \sim \begin{cases} \text{I } x^2-x-2 > 0 \rightarrow x < -1 \text{ or } x > 2 \\ \text{II } x^2-1 > 0 \rightarrow x < -1 \text{ or } x > 1 \end{cases}$

$D_f = (-\infty, -1) \cup (2, \infty)$

$f(x) = \sqrt{x^2+ax-x^2} \quad D_f = [-2, 6] \quad a+b=?$

$x^2+ax-x^2 \geq 0 \rightarrow ax \geq 0 \rightarrow a \geq 0 \rightarrow a = \frac{1}{x} \rightarrow x^2 - \frac{1}{x}x + x \geq 0$

$\Delta = \frac{1}{x^2} - 4(-1)(x) = \frac{4x^3+1}{x^2} \rightarrow \begin{cases} x_1 = \frac{1}{x} + \frac{1}{x} = -2 \\ x_2 = \frac{1}{x} - \frac{1}{x} = 0 \end{cases}$

$a+b = -\frac{1}{x} + \frac{1}{x} = 0$

$f(x) = \begin{cases} x^2-2 & x \geq 1 \\ x^2+3 & x < 1 \end{cases}$

$g(x) = \sqrt{f(x)-x} \rightarrow f(x)-x \geq 0 \rightarrow \begin{cases} x^2-2-x \geq 0 & x \geq 1 \\ x^2+3-x \geq 0 & x < 1 \end{cases}$

$D_g = [-3, \infty)$

$f(x) = x^2 - \epsilon x \quad \frac{x\sqrt{x}}{x} = \sqrt{x} \rightarrow f(\sin \frac{\pi}{4} + \cos \frac{\pi}{4}) = ?$

$f(\sin \frac{\pi}{4}) = x(\sin \frac{\pi}{4})(\sin \frac{\pi}{4} - \epsilon) \Rightarrow \frac{-\epsilon + \sqrt{x}}{x}$

$f(\cos \frac{\pi}{4}) = x(\cos \frac{\pi}{4})(\cos \frac{\pi}{4} - \epsilon) \Rightarrow -1$

$$f(n) = \frac{n+1}{n-1} \quad f(f(a)) = a \quad ? \text{ را در این صورت}$$

$$f(a) = \frac{a+1}{a-1} \rightarrow f\left(\frac{a+1}{a-1}\right) = a \rightarrow \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = a \rightarrow \text{با این روش می توانیم برقرار بگذاریم}$$

$$D_f = \mathbb{R} - \{1\}$$

$$f(n) = n^2 - \varepsilon n \quad g(n) = \sqrt{n^2 - 1} \quad f(g(a)) = -\varepsilon \quad a = ?$$

$$g(a) = \sqrt{a^2 - 1} \rightarrow f(\sqrt{a^2 - 1}) = (\sqrt{a^2 - 1})^2 - \varepsilon(\sqrt{a^2 - 1}) = -\varepsilon$$

$$(\sqrt{a^2 - 1})^2 - \varepsilon(\sqrt{a^2 - 1}) + \varepsilon = 0 \rightarrow (\sqrt{a^2 - 1} - \varepsilon)^2 = 0 \rightarrow \boxed{a = \pm \sqrt{\varepsilon}}$$

$$f(n+r) = n^2 - an + 9 \quad f(\varepsilon) \times f(-\varepsilon) = ? \quad r \times v \times \Rightarrow \boxed{14\varepsilon} \text{ جواب}$$

$$n+r = \varepsilon \rightarrow \boxed{n = 1} \rightarrow f(\varepsilon) = r$$

$$n+r = -\varepsilon \rightarrow \boxed{n = -9} \rightarrow f(-\varepsilon) = vr$$

$$f(\varepsilon \cos n + a \cos n) = \tan n \quad f(r) = ? \quad \frac{r\omega}{1\omega} = \frac{v}{r}$$

$$\varepsilon \cos n + a \cos n = r\omega \rightarrow \cos n (\varepsilon \cos n + a) = r\omega, a \xrightarrow{\cos n = \frac{1}{r}} \frac{1}{r} (\varepsilon + a) = \frac{v}{r}$$

$$f(r, a) = \tan n \rightarrow \tan n \rightarrow \boxed{r} \text{ جواب}$$

$$\boxed{\frac{v}{r} = \frac{v}{r}}$$