

الف - $f(x) = \sqrt{\frac{x-1}{x}} - \frac{x}{x-1}$ $\frac{x-1}{x} - \frac{x}{x-1} = \frac{(x-1)^2 - x^2}{x(x-1)} = \frac{x^2 - 2x + 1 - x^2}{x(x-1)} = \frac{-2x + 1}{x(x-1)}$ $\frac{1-2x}{x(x-1)} > 0$ $1-2x > 0 \rightarrow x < \frac{1}{2}$ $x < \frac{1}{2}$ $D_f: (-\infty, 0) \cup [\frac{1}{2}, 1)$	ب - $f(x) = \frac{1}{x+1} - \frac{x}{x-1}$ $\frac{1}{x+1} - \frac{x}{x-1} = \frac{x-1-x(x+1)}{(x+1)(x-1)} = \frac{x-1-x^2-x}{(x+1)(x-1)} = \frac{-x^2-2x-1}{(x+1)(x-1)}$ $\frac{-x^2-2x-1}{(x+1)(x-1)} > 0$ $x^2+2x+1 < 0$ $(x+1)^2 < 0$ $x \neq -1$ $D_f: (-\infty, -1) \cup (-1, 1)$
الف - $f(x) = \sqrt{((\frac{1}{x})^x - 9)(3x - x^2)}$ $((\frac{1}{x})^x - 9) = 0 \rightarrow (\frac{1}{x})^x = 9 \rightarrow x^{-x} = 9 \rightarrow x^{-x} = 3^2$ $x^{-x} = 3^2$ $x = 3$ $x = -3$ $3x - x^2 = 0 \rightarrow x(3-x) = 0 \rightarrow x = 0 \text{ or } x = 3$ $x = 3$ $D_f: (-\infty, -3] \cup [3, +\infty)$	ب - $f(x) = \sqrt{x-1} + \sqrt{y+1} = 3$ $\sqrt{x-1} + \sqrt{y+1} = 3$ $\sqrt{x-1} \leq 3 \rightarrow x-1 \leq 9 \rightarrow x \leq 10$ $\sqrt{y+1} \leq 3 \rightarrow y+1 \leq 9 \rightarrow y \leq 8$ $D_f: [1, 10] \times [0, 8]$
$f(x) = \log_x(x^2 - x - 2)$ $\log_x(x^2 - x - 2) > 0$ $x^2 - x - 2 > 1$ $x^2 - x - 3 > 0$ $(x-3)(x+1) > 0$ $x < -1$ or $x > 3$ $D_f: (-\infty, -1) \cup (3, +\infty)$	III) $x^2 - 1 > 0 \rightarrow x > 1$ or $x < -1$ $-1 < x < 1$ $D_f: (-\infty, -1) \cup (1, +\infty)$
$f(x) = \sqrt{-x^2 + 2x + 3}$ $D_f: [-1, 3]$ $a+b = 2$ if $x = -1 \rightarrow -1 - 2a + 3 = 0 \rightarrow -2a + 2 = 0 \rightarrow a = 1$ $a = 1$ $b = 1$ $D_f: [-1, 3]$	$f(x) = \sqrt{-x^2 + 2x + 3}$ $D_f: [-1, 3]$ $a+b = 2$ if $x = 3 \rightarrow -9 + 6 + 3 = 0$ $a = 3$ $b = -1$ $D_f: [-1, 3]$
$f(x) = \sqrt{f(x) - x}$ $D_f: ?$ $f(x) - x \geq 0$ $f(x) = x$ $D_f: [-3, +\infty)$	$f(x) = \sqrt{f(x) - x}$ $D_f: ?$ $f(x) - x \geq 0$ $f(x) = x$ $D_f: [-3, +\infty)$

$f(x) - x \geq 0 \rightarrow x^2 - 2x + 3 - x \geq 0 \rightarrow x^2 - 3x + 3 \geq 0$

$x^2 - 3x + 3 \geq 0$

$D_f: [-3, +\infty)$

$$f(x) = x^2 - x$$

$$f\left(\sin \frac{\pi}{4} + \cos \frac{\pi}{4}\right) = ?$$

$$\begin{aligned} f(a) &= \left(\sqrt{\frac{2}{2}} + 1\right)^2 - \left(\sqrt{\frac{2}{2}} + 1\right) \\ &= \frac{2}{2} + 1 + \sqrt{2} - \sqrt{2} - 1 \\ &= \frac{2}{2} - 1 = -\frac{1}{2} \end{aligned}$$

$$f(x) = \frac{x+1}{x-1}$$

$$f(f(a)) = a$$

پیدا کنید

$$f(a) = \frac{a+1}{a-1}$$

$$f(f(a)) = \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1}$$

$$\frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = a$$

$$\frac{a+1+a-1}{a+1-a-1} = a$$

$$\frac{2a}{a-2} = a - 2a = -a$$

با توجه به معادله بدست آمده ضابطه ۲ برای معادله بدست آمده به کار می آید و به دست می آید که $a \in \mathbb{R} - \{2\}$

$$f(x) = x^2 - x$$

$$g(x) = \sqrt{x^2 - 1}$$

$$f(g(a)) = -1$$

$$a = ?$$

$$g(a) = \sqrt{a^2 - 1}$$

$$f(g(a)) = (a^2 - 1)^2 - \sqrt{a^2 - 1} = a^2 - 1 - \sqrt{a^2 - 1}$$

$$a^2 - 1 - \sqrt{a^2 - 1} = -1$$

$$(a^2 - 1) - \sqrt{a^2 - 1} + 1 = 0$$

$$x^2 - x + 1 = 0$$

$$(x-1)^2 = 0 \rightarrow x = 1$$

$$\sqrt{a^2 - 1} = 1 \quad (1)$$

$$a^2 - 1 = 1$$

$$a^2 = 2 \rightarrow a = \pm\sqrt{2}$$

$$f(x+3) = x^2 - 2x + 1$$

$$f(1) \times f(-1) = ?$$

$$x+3 = 1 \rightarrow x = -2$$

$$f(1) = 1 - 2 + 1 = 0$$

$$x+3 = -1 \rightarrow x = -4$$

$$f(-1) = 1 - 2 + 1 = 0$$

$$f(1) \times f(-1) = 0 \times 0 = 0$$

با توجه به معادله بدست آمده

با توجه به معادله بدست آمده

$$f(\cos^2 x + \sin^2 x) = \tan^2 x$$

$$f(1) = ?$$

$$\tan^2 x = \frac{\sin^2 x}{\cos^2 x}$$

$$\sin^2 x + \cos^2 x = 1$$

$$\sin^2 x = 1 - \cos^2 x = \frac{1}{2}$$

$$f(1) = \frac{1}{2} = \frac{1}{2}$$

$$f(\cos^2 x + \sin^2 x) = \frac{1}{2}$$

$$f(\cos^2 x + \sin^2 x) = \frac{1}{2} = 0$$

$$\cos^2 x + \sin^2 x - 1 = 0$$

$$(\cos x + 1)(\cos x - 1) = 0$$

$$\cos x = \frac{1}{2} = -\frac{1}{2}$$

$$\cos x = \frac{1}{2} = \frac{1}{2}$$

با توجه به معادله بدست آمده