

الف) $f(n) = \sqrt{\frac{n-1}{n}} - \frac{n}{n-1}$

$n \neq 0$ (1)

$n \neq 1$ (2)

$\frac{\frac{n-1}{n}}{\frac{n-1}{n}} - \frac{n}{n-1} = \frac{n+1-2n}{n(n-1)} = \frac{-n+1}{n(n-1)}$

(1) $\cap$ (2) $\cap$ (3) = (3)

$= D_f = (-\infty, 0) \cup [\frac{1}{2}, 1)$

$\frac{-n+1}{n(n-1)} \geq 0 \rightarrow \begin{cases} -n+1=0 \rightarrow n=1 \\ n=0 \\ n=1 \end{cases}$

$\frac{0}{+} \frac{1}{-} \frac{1}{+} \rightarrow (-\infty, 0) \cup [\frac{1}{2}, 1)$ (3)

ب) $f(n) = \frac{\frac{1}{n+1} - \frac{2}{n}}{\frac{2}{n-1} + \frac{1}{n+2}}$

$\frac{\frac{1}{n+1} - \frac{2}{n}}{\frac{2}{n-1} + \frac{1}{n+2}} = \frac{\frac{n+2-2n-2}{(n+1)n}}{\frac{2n+2+n-1}{(n-1)(n+2)}} = \frac{-n}{(n-1)(n+2)} \neq 0$

$\rightarrow \frac{-n}{(n-1)(n+2)} = 0 \rightarrow n=0$ (3)

$\rightarrow D_f = (1) \cap (2) = \mathbb{R} - \{0, -2, \frac{2}{3}\}$

ج) $f(n) \sqrt{((\frac{1}{2})^n - 9)(22 - 4^n)} \quad ((\frac{1}{2})^n - 9)(22 - 4^n) \geq 0$

$2 \cdot 3^n = 2^2 \rightarrow n=2$

$2 = 2^n \rightarrow n=1$

$\frac{1}{+} \frac{2}{-} \frac{2}{+} \rightarrow D_f = (-\infty, 1) \cup [2, \infty)$ (1, 5)

د) $\sqrt[n]{n-1} + \sqrt[n]{n+1} = 2 \rightarrow \sqrt[n]{n+1} = 2 - \sqrt[n]{n-1} \rightarrow y+1 = 9 - 9\sqrt[n]{n-1} - 9\sqrt[n]{n-1}$
 $\rightarrow y = \sqrt[n]{n-1} - 9\sqrt[n]{n-1} + 1 \rightarrow n \geq 1$

$\sqrt[n]{y+1} = 3 - \sqrt[n]{n-1} \rightarrow \text{باید همواره مثبت باشد} \rightarrow \sqrt[n]{n-1} \leq 3 \rightarrow n-1 \leq 27 \rightarrow n \leq 28$

$D_f = [1, 28]$

$$f(n) = \frac{\log_e (n^2 - n + 1)}{\sqrt{n^2 - 1} + 1} \rightarrow n^2 - n + 1 > 0 \rightarrow (n - \frac{1}{2})(n - \frac{1}{2}) > 0 \rightarrow \frac{1}{2} - \frac{1}{2} = 0 \quad (1)$$

$$n^2 - 1 > 0 \rightarrow (n - 1)(n + 1) > 0 \rightarrow \frac{1}{2} - \frac{1}{2} = 0 \quad (2)$$

$$P_f = (1) \cap (2) = (-\infty, -1) \cup (2, \infty) \quad \text{✓}$$

$$f(n) = \sqrt{r \tan n - n^2} \quad P_f = [-r, b] \rightarrow \text{محدوده تعریف}$$

$$r \tan n - n^2 \geq 0 \rightarrow r \tan n \geq n^2$$

$$n = -r \rightarrow -r \tan r + r^2 = 0 \rightarrow -1 = \tan r \rightarrow r = -\frac{\pi}{2}$$

$$-n^2 - \frac{1}{r} n + r = 0 \rightarrow n^2 + \frac{1}{r} n - r = 0 \rightarrow r^2 n + n - r = 0 \rightarrow n^2 - n - r = 0 \rightarrow (n + r)(n - r) = 0$$

$$\rightarrow n = -\frac{r}{2} = -r, \quad n = \frac{r}{2} = 1 \Rightarrow b$$

$$\rightarrow a = -\frac{r}{2} > b = \frac{r}{2} \rightarrow a + b = \frac{r}{2} - \frac{r}{2} = 0 \quad \text{✓}$$

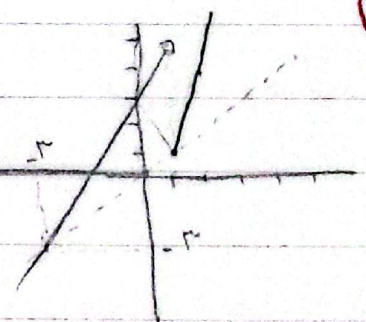
$$f(n) = \begin{cases} r n - r & n \geq 1 \\ r n + r & n < 1 \end{cases}$$

$$g(n) = \sqrt{f(n) - n} \rightarrow f(n) - n \geq 0 \rightarrow f(n) \geq n$$

$$\rightarrow P_g = [-r, +\infty)$$

$$r n + r = n \rightarrow n = -r$$

آفرین نه مثل کسیک نت



$$f(n) = r n^2 - 4 n$$

$$f(\sin \frac{\pi}{4} + \cos 2\pi) = f(\frac{\sqrt{2}}{2} + 1) = 2(\frac{\sqrt{2}}{2} + 1)^2 - 4(\frac{\sqrt{2}}{2} + 1) = 2(\frac{2}{2} + 1 + \sqrt{2}) - 4(\frac{\sqrt{2}}{2} + 1)$$

$$= 2\sqrt{2} - 4 = \frac{2\sqrt{2}}{2} + 2 + 2\sqrt{2} - 2\sqrt{2} - 4 = 1 + 2 - 4 = -1 \quad \text{✓}$$

$$f(n) = \frac{n+1}{n-1} \quad f(f(n)) = a$$

(v)

$$f(n) = \frac{n+1}{n-1} \rightarrow \frac{\frac{n+1}{n-1} + 1}{\frac{n+1}{n-1} - 1} = a \rightarrow \frac{\frac{n+1+a-1}{n-1}}{\frac{n+1-a+1}{n-1}} = \left(\frac{\frac{a}{n-1}}{\frac{2}{n-1}} \right) = a = a \quad \checkmark$$

$$a \neq 1 \rightarrow n \neq 1 \rightarrow \mathbb{R} - \{1\}$$

(v)

$$f(n) = n^r - \varepsilon n \quad g(n) = \sqrt{n^r - 1} \quad f(g(n)) = -\varepsilon$$

(1)

$$f(g(n)) = -\varepsilon \rightarrow (g(n))^r - \varepsilon g(n) + \varepsilon = 0 \rightarrow (g(n) - \varepsilon)^r = 0 \rightarrow g(n) = \varepsilon$$

$$\rightarrow \sqrt{n^r - 1} = \varepsilon \rightarrow n^r - 1 = \varepsilon^2 \rightarrow n^r = \varepsilon^2 + 1 \rightarrow n = \pm \sqrt{\varepsilon^2 + 1}$$

(v)

$$f(n, r) = n^r - \varepsilon n + \gamma$$

(7)

$$\rightarrow f(n) = (n, r)$$

$$\rightarrow f(\varepsilon) \times f(r) \rightarrow \begin{array}{l} n+r = \varepsilon \rightarrow n=1 \rightarrow f(\varepsilon) = 1 - \varepsilon + \gamma = \gamma \\ n+r = -r \rightarrow n=-\gamma \rightarrow f(-r) = r^r + r\varepsilon + \gamma = \gamma \end{array} \quad \boxed{\gamma \times \gamma = 1 \text{ if } \gamma = 1}$$

(v)

$$f(\varepsilon \cos n + \omega \cos n) = \tan^r n$$

$$\begin{array}{c} \omega \neq r \\ \varepsilon \neq 1 \end{array}$$

$$\varepsilon \times 1 \neq$$

(1)

$$f(r/a) = ?$$

$$\varepsilon \neq \varepsilon$$

$$\varepsilon \cos^r n + \omega \cos n = \frac{\omega}{\varepsilon} \rightarrow \varepsilon \cos^r n + \omega \cos n = -\frac{\omega}{\varepsilon} = 0 \rightarrow \varepsilon \cos^r n + \omega \cos n - \omega = 0$$

$$\rightarrow \cos^r n + \omega \cos n = 0 \rightarrow (\cos n + 1)(\cos n - 1) = 0 \rightarrow \cos n = \frac{-1 \pm \sqrt{1 - \frac{\omega^2}{\varepsilon^2}}}{2} = \frac{-1 \pm \sqrt{1 - \frac{\omega^2}{\varepsilon^2}}}{2}$$

$$\rightarrow \cos n = \frac{1}{\varepsilon} = \frac{1}{\varepsilon} \quad \text{if } \varepsilon = 1$$

$$\rightarrow \cos n = \frac{1}{\varepsilon} \rightarrow 1 + \tan^r n = \frac{1}{\cos^r n} = \frac{1}{\frac{1}{\varepsilon^r}} = \varepsilon^r \rightarrow 1 + \tan^r n = \varepsilon^r \rightarrow \tan^r n = \varepsilon^r - 1$$

$$\rightarrow f(r/a) = 1 \quad \text{if } \varepsilon = 1 \rightarrow \tan n = \pm \sqrt{\varepsilon^r - 1} \quad \text{if } \tan n = \sqrt{\varepsilon^r - 1}$$

(1)

ب-۲) $\sqrt[n]{x-1} + \sqrt[n]{y+1} = 3$, $\underbrace{\sqrt[n]{y+1}}_{\text{همواره مثبت}} = \underbrace{3 - \sqrt[n]{x-1}}_{\text{همواره مثبت!}} \rightarrow 3 - \sqrt[n]{x-1} \geq 0 \rightarrow \sqrt[n]{x-1} \leq 3 \rightarrow x \leq 125$

$\downarrow$
 $x-1 \geq 0 \rightarrow x \geq 1$ I

I, II $\rightarrow 1 \leq x \leq 125 \rightarrow Df = [1, 125]$

$f(x \cdot \sin^n + a \cdot \sin) = f(x, a) \rightarrow x \cdot \sin^n + a \cdot \sin = 3, a \rightarrow x \cdot \sin^n + a \cdot \sin - \frac{a}{x} = 0$ - 1

$C \cdot \sin = \frac{-a \pm \sqrt{a^2 + 4x}}{2} \rightarrow C \sin = \frac{-a+4}{2} = \frac{1}{2}$
 $\hookrightarrow C \cdot \sin = \frac{-a-4}{2} = -\frac{a}{2} \times C \cdot \sin \geq -$

محدود: $C \cdot \sin \frac{\pi}{2} = \frac{1}{2}$, $C \cdot \sin \frac{3\pi}{2} = \frac{1}{2}$ $\rightarrow f(x, a) = 3$

$\downarrow$ $\downarrow$
 $\tan \frac{\pi}{2} = (\sqrt{x})^2 = 3$ $\tan \frac{3\pi}{2} = (-\sqrt{x})^2 = 3$