

1) الف)  $f(n) = \sqrt{\frac{n-1}{n} - \frac{n}{n-1}}$

$n \neq 0$  (1)

$n \neq 1$  (2)

$\frac{\frac{n-1}{n}}{n-1} - \frac{n}{n-1} = \frac{n-1 - n^2}{n(n-1)} = \frac{-n^2 + n - 1}{n(n-1)}$

(1)  $\cap$  (2)  $\cap$  (3) = (3)

$= P_f = (-\infty, 0) \cup [\frac{1}{e}, 1)$

$\frac{-n^2 + n - 1}{n(n-1)} \geq 0$   
 $-n^2 + n - 1 = 0 \rightarrow n = \frac{1 \pm \sqrt{5}}{2}$   
 $n = 0$   
 $n = 1$

$\frac{0}{+} \frac{1}{-} \frac{1}{+} \frac{1}{-} \rightarrow (-\infty, 0) \cup [\frac{1}{e}, 1)$  (3)

ب)  $f(n) = \frac{\frac{1}{n+1} - \frac{2}{n}}{\frac{2}{n-1} + \frac{1}{n+2}}$

$\frac{\frac{1}{n+1}}{\frac{2}{n-1}} - \frac{\frac{2}{n}}{\frac{1}{n+2}} = \frac{n-1}{2(n+1)} - \frac{2(n+2)}{n+2} = \frac{n-1-4(n+2)}{2(n+1)(n+2)} = \frac{-3n-7}{2(n+1)(n+2)} \neq 0$

$-3n-7=0 \rightarrow n = -\frac{7}{3}$  (3)

$\rightarrow P_f = (1) \cap (2) = \mathbb{R} - \{-1, 0, 1, -\frac{7}{3}\}$

ج) الف)  $f(n) \sqrt{((\frac{1}{e})^n - 9)(22 - 4^n)} \quad ((\frac{1}{e})^n - 9)(22 - 4^n) \geq 0$  (1)

$\frac{1}{e^n} = 9 \rightarrow n = -2$

$22 = 4^n \rightarrow n = \log_4 22$

$\frac{-1}{+} \frac{0}{-} \frac{0}{+} \frac{1}{-} \rightarrow P_f = (-\infty, -2] \cup [\log_4 22, \infty)$

ب)  $\sqrt{n-1} + \sqrt{y+1} = 5 \rightarrow \sqrt{y+1} = 5 - \sqrt{n-1} \rightarrow y+1 = 25 + n-1 - 9\sqrt{n-1}$   
 $\rightarrow y = \sqrt{n-1} - 9\sqrt{n-1} + 1 \rightarrow n \geq 1$

$$f(n) = \frac{\log_e (n^r - n - r)}{\sqrt{n^r - 1} + 1} \rightarrow n^r - n - r \geq 0 \rightarrow (n - r)(n + 1) \geq 0 \rightarrow \frac{1}{n - r} - \frac{1}{n + 1} \geq 0 \quad (1)$$

$$n^r - 1 \geq 0 \rightarrow (n - 1)(n + 1) \geq 0 \rightarrow \frac{1}{n - 1} - \frac{1}{n + 1} \geq 0 \quad (2)$$

$$P_f = (1) \cap (2) = (-\infty, -1] \cup (r, \infty)$$

$$f(n) = \sqrt{r n + r} \quad P_f = [a, b] \rightarrow \text{range of } f$$

$$r n + r \geq 0 \rightarrow n \geq -\frac{r}{r} = -1$$

$$n = -1 \rightarrow -r - r + r = 0 \rightarrow -1 = r \rightarrow r = -1$$

$$-n^r - \frac{1}{r} n + r = 0 \rightarrow n^r + \frac{1}{r} n - r = 0 \rightarrow r n^r + n - r = 0 \rightarrow n^r - n - 1 = 0 \rightarrow (n + r)(n - r) = 0$$

$$\rightarrow n = -\frac{r}{r} = -1, \quad n = \frac{r}{r} = 1 \Rightarrow a = b$$

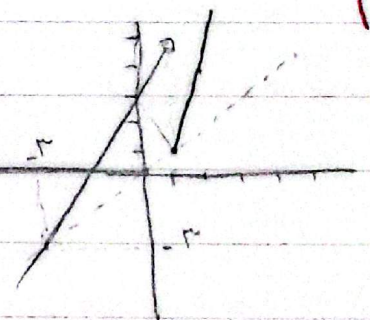
$$\rightarrow a = -1, b = 1 \rightarrow a + b = -1 + 1 = 0$$

$$f(n) = \begin{cases} r n - r & n \geq 1 \\ r n + r & n \leq 1 \end{cases}$$

$$g(n) = \sqrt{f(n)} - n \rightarrow f(n) - n \geq 0 \rightarrow f(n) \geq n$$

$$\rightarrow P_g = [-r, +\infty)$$

$$r n + r = n \rightarrow n = -r$$



$$f(n) = r n^r - r n$$

$$f\left(\sin \frac{\pi}{r} + \cos r \pi\right) = f\left(\frac{\sqrt{r}}{r} + 1\right) = r\left(\frac{\sqrt{r}}{r} + 1\right)^r - r\left(\frac{\sqrt{r}}{r} + 1\right) = r\left(\frac{r}{r} + 1 + \sqrt{r}\right) - r\sqrt{r} - r = \frac{r}{r} + r + r\sqrt{r} - r\sqrt{r} - r = 1 + r - r = -r = -\frac{1}{r}$$

$$f(n) = \frac{n+1}{n-1} \quad f(f(n)) = n \quad (v)$$

$$\rightarrow f(n) = \frac{n+1}{n-1} \rightarrow \frac{\frac{n+1}{n-1} + 1}{\frac{\frac{n+1}{n-1} - 1} = n \rightarrow \frac{\frac{n+1+n-1}{n-1}}{\frac{n+1-n-1}{n-1}} = \left( \frac{\frac{2n}{n-1}}{\frac{-1}{n-1}} \right) = n = n \quad \checkmark$$

$$n \neq 1 \rightarrow n \neq 1 \rightarrow \mathbb{R} - \{1\}$$

$$f(n) = n^r - \epsilon n \quad g(n) = \sqrt{n^r - 1} \quad f(g(n)) = -\epsilon \quad (u)$$

$$f(g(n)) = -\epsilon \rightarrow (g(n))^r - \epsilon g(n) + \epsilon = 0 \rightarrow (g(n) - \epsilon)^r = 0 \rightarrow g(n) = \epsilon$$

$$\rightarrow \sqrt{n^r - 1} = \epsilon \rightarrow n^r - 1 = \epsilon^2 \rightarrow n^r = \epsilon^2 + 1 \rightarrow n = \pm \sqrt[r]{\epsilon^2 + 1}$$

$$f(n, r) = n^r - \epsilon n + \gamma \quad (7)$$

$$\rightarrow f(n) = (n, r)$$

$$\rightarrow f(\epsilon) \times f(r) \rightarrow \begin{array}{l} n+r=\epsilon \rightarrow n=1 \rightarrow f(\epsilon) = 1 - \epsilon + \gamma = \gamma \\ n+r=r \rightarrow n=-\epsilon \rightarrow f(r) = r^r + r\epsilon + \gamma = \gamma r \end{array} \quad \gamma \times \gamma r = 1 \epsilon f$$

$$f(\epsilon \cos n + \omega \cos n) = \tan^r n$$

$$\begin{array}{l} \omega \gamma / r \\ \epsilon \Lambda \end{array}$$

$$\epsilon \times 1 \epsilon$$

$$(1)$$

$$f(r/\Lambda) = ?$$

$$\epsilon \times \epsilon \times \epsilon$$

$$\epsilon \cos^r n + \omega \cos n = \frac{\omega \gamma}{r} \rightarrow \epsilon \cos^r n + \omega \cos n = -\frac{\gamma}{r} = 0 \rightarrow \Lambda \cos^r n + \omega \cos n - \gamma = 0$$

$$\rightarrow \cos^r n + \omega \cos n - \gamma = 0 \rightarrow (\cos n + \epsilon)(\cos n - \epsilon) = 0 \rightarrow \cos n = \frac{-\epsilon}{\Lambda} = -\frac{\gamma}{r}$$

$$\rightarrow \cos n = \frac{\gamma}{\Lambda} = \frac{1}{r} \quad \checkmark$$

$$\rightarrow \cos n = \frac{1}{r} \rightarrow 1 + \tan^r n = \frac{1}{\cos^r n} = \frac{1}{\frac{1}{r^r}} = r^r \rightarrow 1 + \tan^r n = r^r \rightarrow \tan^r n = r^r - 1 \rightarrow \tan n = \sqrt[r]{r^r - 1} \quad -1 < \cos n < 1$$

$$\rightarrow f(r/\Lambda) = 1 \epsilon$$