

پایه نهم ریاضی

الف) $f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}} \geq 0 \Rightarrow \frac{(x-1)(x-1) - x^2}{x(x-1)} \geq 0 \Rightarrow \frac{x^2 - 2x + 1 - x^2}{x(x-1)} \geq 0 \Rightarrow \frac{1-2x}{x(x-1)} \geq 0$
 $D_f = (-\infty, 0) \cup [\frac{1}{2}, \infty)$

ب) $f(x) = \frac{\frac{1}{x+1} - \frac{x}{x-1}}{\frac{x}{x-1} + \frac{1}{x+1}} \Rightarrow \frac{\frac{x - 2x - 2}{x(x+1)}}{\frac{x^2 + x + x - 1}{(x-1)(x+1)}} = \frac{-x-2}{x^2+x-1}$
 $D_f = R - \left\{ -\frac{1}{2}, -\frac{1}{x} \right\}$

الف) $f(x) = \sqrt{((\frac{1}{x})^2 - 9)(x^2 - 9)} \Rightarrow \begin{cases} (\frac{1}{x})^2 - 9 = 0 \Rightarrow (\frac{1}{x})^2 = 9 \Rightarrow (\frac{1}{x})^2 = 3^2 \Rightarrow x^2 = \frac{1}{9} \Rightarrow x = \pm \frac{1}{3} \\ x^2 - 9 = 0 \Rightarrow x^2 = 9 \Rightarrow x = \pm 3 \end{cases}$
 $D_f = (-\infty, -3] \cup [\frac{1}{3}, \infty)$

ب) $\sqrt{x-1} + \sqrt{y+1} = 3 \Rightarrow \sqrt{y+1} = 3 - \sqrt{x-1} \Rightarrow \begin{cases} \sqrt{x-1} \geq 0 \Rightarrow x-1 \geq 0 \Rightarrow x \geq 1 \\ \sqrt{y+1} \geq 0 \Rightarrow 3 - \sqrt{x-1} \geq 0 \Rightarrow \sqrt{x-1} \leq 3 \Rightarrow x-1 \leq 9 \Rightarrow x \leq 10 \end{cases}$
 $D_f = [1, 10]$

$f(x) = \frac{\log_e (x^2 - x - 2)}{\sqrt{x^2 - 1} + 1} \Rightarrow \begin{cases} \log_e (x^2 - x - 2) \geq 0 \Rightarrow (x^2 - x - 2) \geq 0 \Rightarrow (x-2)(x+1) \geq 0 \\ \sqrt{x^2 - 1} + 1 \neq 0 \Rightarrow \sqrt{x^2 - 1} \neq -1 \end{cases}$
 $D_f = (-\infty, -1) \cup (2, \infty)$

$f(x) = \sqrt{x^2 + ax - x^2} \Rightarrow x^2 + ax - x^2 \geq 0 \Rightarrow x^2 - x^2 + ax \geq 0 \Rightarrow ax \geq 0 \Rightarrow a \geq 0$
 $\hookrightarrow \sqrt{x^2 + \frac{1}{4}x - x^2} \Rightarrow x^2 + \frac{1}{4}x - x^2 \geq 0 \Rightarrow x^2 + x - 4 \geq 0 \Rightarrow (x+2)(x-4) \geq 0$
 $D_f = [-2, 4]$

$f(x) = \begin{cases} x^2 - 2 & x \geq 1 \\ x^2 + 2 & x < 1 \end{cases}$
 $g(x) = \sqrt{f(x) - x} \Rightarrow f(x) - x \geq 0 \Rightarrow \begin{cases} x \geq 1 \Rightarrow x^2 - 2 - x \geq 0 \Rightarrow x^2 - x - 2 \geq 0 \Rightarrow (x-2)(x+1) \geq 0 \Rightarrow x \geq 2 \\ x < 1 \Rightarrow x^2 + 2 - x \geq 0 \Rightarrow x^2 - x + 2 \geq 0 \Rightarrow (x-1)^2 + 1 \geq 0 \Rightarrow x < 1 \end{cases}$
 $D_f = [-2, \infty)$

$$f(x) = x^n - x$$

$$f(\sin \frac{\pi}{4} + \cos \frac{\pi}{4}) \rightarrow \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}, \cos \frac{\pi}{4} = 1 \Rightarrow f(\frac{\sqrt{2}}{2} + 1)$$

$$\rightarrow r((\frac{\sqrt{2}}{2} + 1)^2) - f(\frac{\sqrt{2}}{2} + 1) \Rightarrow r(\frac{2}{2} + 1 + \sqrt{2}) - \frac{\sqrt{2}}{2} + 1 = 1 + \sqrt{2} - \frac{\sqrt{2}}{2} + 1 = 2 + \frac{\sqrt{2}}{2} \Rightarrow \boxed{-\frac{1}{2}}$$

$$f(x) = \frac{x+1}{x-1}$$

$$f(f(a)) = a \Rightarrow f(a) = \frac{a+1}{a-1} \Rightarrow f(f(a)) = \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = \frac{a+1+a-1}{a+1-a-1} = \frac{2a}{a-2} = a \checkmark$$

$$f(a) = \frac{a+1}{a-1} \rightarrow D_f: \mathbb{R} - \{1\}$$

$$f(a) \neq 1 \Rightarrow \frac{a+1}{a-1} \neq 1 \Rightarrow a+1 \neq a-1 \Rightarrow 1 \neq -1 \text{ صحیح } \checkmark \Rightarrow \mathbb{R} - \{1\}$$

به ازای همه اعداد حقیقی
به جز 1

$$f(x) = x^2 - x$$

$$g(x) = \sqrt{x^2 - 1} \xrightarrow{x \geq 1} (\sqrt{x^2 - 1})^2 - f(\sqrt{x^2 - 1}) = -x \Rightarrow x^2 - 1 - f(\sqrt{x^2 - 1}) = -x \Rightarrow \frac{x^2 - 1}{x^2} - f(\frac{\sqrt{x^2 - 1}}{x}) + x = 0$$

$$f(g(a)) = -x$$

$$\Rightarrow x^2 - f(x) + x = 0 \Rightarrow (x-2)^2 = 0 \Rightarrow x=2 \Rightarrow \sqrt{x^2 - 1} = 2 \Rightarrow x^2 - 1 = 4 \Rightarrow x^2 = 5 \Rightarrow x = \pm \sqrt{5}$$

$$g(x) = \sqrt{x^2 - 1} \Rightarrow x^2 - 1 \geq 0 \Rightarrow x^2 \geq 1 \Rightarrow \begin{cases} x \geq 1 \\ x \leq -1 \end{cases} \Rightarrow \begin{cases} a \geq \sqrt{5} \\ a \leq -\sqrt{5} \end{cases} \Rightarrow \sqrt{a^2 - 1} = \sqrt{5} = 2 \checkmark$$

$$f(x+2) = x^2 - 2x + 4$$

$$f(x) \cdot f(-x) \Rightarrow \begin{cases} x+2 = x \Rightarrow x=2 \Rightarrow 1-2+4 = 3 = f(x) \xrightarrow{x} \sqrt{x^2 - 2} = \sqrt{4-2} = \sqrt{2} \\ x+2 = -x \Rightarrow x=-2 \Rightarrow 4-4+4 = 4 = f(-x) \xrightarrow{x} \sqrt{x^2 - 2} = \sqrt{4-2} = \sqrt{2} \end{cases}$$

از این روش سنجش

$$f(\cos x + \sin x) = \tan x \xrightarrow{*} \frac{\sin x}{\cos x} = \frac{1 - \cos x}{\cos x}$$

$$f(x, a) = 3 \Rightarrow f \cos x + \sin x = 2 \cos x \Rightarrow \cos x + \sin x = 2 \cos x \Rightarrow \sin x = \cos x \Rightarrow \tan x = 1 \Rightarrow x = \frac{\pi}{4}$$

$$\hookrightarrow \cos x + 1 \cdot \cos x - \sin x = 0$$

$$\hookrightarrow \cos^2 x + 1 \cdot \cos x - \sin^2 x = 0$$

$$(\cos x + 1)(\cos x - \frac{1}{2}) = 0$$

$$\Rightarrow \cos x = -\frac{1}{2} \text{ و } \cos x = \frac{1}{2}$$

$$\textcircled{1} \rightarrow -1 \leq \cos x \leq 1 \Rightarrow \cos x = \frac{1}{2}$$

در اینجا به عنوان قبول است

$$\hookrightarrow \frac{1 - \frac{1}{2}}{\frac{1}{2}} = \frac{\frac{1}{2}}{\frac{1}{2}} = 1 \Rightarrow \boxed{3}$$

①