

الف) $f(x) = \sqrt{\frac{x-1}{x}} - \frac{x}{x-1} \geq 0 \Rightarrow \frac{(x-1)(x-1) - x^2}{x(x-1)} \geq 0 \Rightarrow \frac{x^2 - 2x + 1 - x^2}{x(x-1)} \geq 0 \Rightarrow \frac{1-2x}{x(x-1)} \geq 0 \Rightarrow \frac{0}{1} \frac{1}{1} \frac{1}{1}$
 $D_f = (-\infty, 0) \cup [1, +\infty)$

ب) $f(x) = \frac{1}{x+1} - \frac{x}{x-1} \Rightarrow \frac{x - x^2 - x^2 - 1}{(x+1)(x-1)} = \frac{-x^2 - x - 1}{(x+1)(x-1)}$
 $D_f = R - \left\{ -1, -\frac{1}{2}, 1 \right\}$

الف) $f(x) = \sqrt{\left(\left(\frac{1}{x}\right)^2 - 9\right)(x^2 - x^2)} \Rightarrow \begin{cases} \left(\frac{1}{x}\right)^2 - 9 = 0 \Rightarrow \left(\frac{1}{x}\right)^2 = 9 \Rightarrow \left(\frac{1}{x}\right)^2 = 3^2 \Rightarrow x^2 = \frac{1}{9} \Rightarrow x = \pm \frac{1}{3} \\ x^2 - x^2 = 0 \Rightarrow x^2 = x^2 \Rightarrow x^2 = x^2 \Rightarrow x = \frac{1}{x} \Rightarrow x = \pm 1 \end{cases}$
 $D_f = (-\infty, -\frac{1}{3}] \cup [\frac{1}{3}, +\infty)$

ب) $\sqrt{x-1} + \sqrt{y+1} = 2 \Rightarrow \sqrt{y+1} = 2 - \sqrt{x-1} \Rightarrow \begin{cases} \sqrt{x-1} \geq 0 \Rightarrow x-1 \geq 0 \Rightarrow x \geq 1 \\ \sqrt{y+1} \geq 0 \Rightarrow 2 - \sqrt{x-1} \geq 0 \Rightarrow \sqrt{x-1} \leq 2 \Rightarrow x-1 \leq 4 \Rightarrow x \leq 5 \\ x \geq 1 \end{cases}$
 $D_f = [1, 5]$

$f(x) = \frac{\log_e(x^2 - x - 1)}{\sqrt{x^2 - 1} + 1} \Rightarrow \begin{cases} \log_e(x^2 - x - 1) \geq 0 \Rightarrow (x^2 - x - 1) \geq 1 \Rightarrow (x-2)(x+1) \geq 0 \Rightarrow x \leq -1 \text{ or } x \geq 2 \\ \sqrt{x^2 - 1} + 1 \neq 0 \Rightarrow \sqrt{x^2 - 1} \neq -1 \text{ (always true)} \\ \sqrt{x^2 - 1} \geq 0 \Rightarrow x^2 - 1 \geq 0 \Rightarrow x^2 \geq 1 \Rightarrow x \leq -1 \text{ or } x \geq 1 \end{cases}$
 $D_f = (-\infty, -1) \cup (2, +\infty)$

$f(x) = \sqrt{x^2 + ax - x^2} \Rightarrow x^2 + ax - x^2 \geq 0 \Rightarrow x^2 - x^2 + ax \geq 0 \Rightarrow ax \geq 0 \Rightarrow a \geq 0$
 $\hookrightarrow \sqrt{x^2 + \frac{1}{4}x - x^2} \Rightarrow x^2 + \frac{1}{4}x - x^2 \geq 0 \Rightarrow x^2 + x - 4 \geq 0 \Rightarrow (x+2)(x-4) \geq 0 \Rightarrow x \leq -2 \text{ or } x \geq 4$
 $D_f = [-2, 4] \rightarrow b = \frac{1}{4} \Rightarrow a+b = -\frac{1}{4} + \frac{1}{4} = 0$

$f(x) = \begin{cases} x^2 - 2 & x \geq 1 \\ x^2 + 2 & x < 1 \end{cases}$
 $g(x) = \sqrt{f(x) - x} \Rightarrow f(x) - x \geq 0 \Rightarrow \begin{cases} x \geq 1 \Rightarrow x^2 - 2 - x \geq 0 \Rightarrow x^2 - x - 2 \geq 0 \Rightarrow (x-2)(x+1) \geq 0 \Rightarrow x \geq 2 \text{ or } x \leq -1 \\ x < 1 \Rightarrow x^2 + 2 - x \geq 0 \Rightarrow x^2 - x + 2 \geq 0 \Rightarrow (x-1)^2 + 1 \geq 0 \Rightarrow \text{always true} \end{cases}$
 $D_f = [-2, +\infty)$

$$f(x) = x^n - x$$

$$f(\sin \frac{\pi}{4} + \cos \frac{\pi}{4}) \rightarrow \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}, \cos \frac{\pi}{4} = 1 \Rightarrow f(\frac{\sqrt{2}}{2} + 1)$$

$$\rightarrow r((\frac{\sqrt{2}}{2} + 1)^2) - f(\frac{\sqrt{2}}{2} + 1) \Rightarrow r(\frac{2}{2} + 1 + \sqrt{2}) - \frac{\sqrt{2}}{2} + 1 = \frac{1}{2} + 1 + \sqrt{2} - \frac{\sqrt{2}}{2} + 1 = \frac{3}{2} + \frac{\sqrt{2}}{2} = \frac{3 + \sqrt{2}}{2}$$

$$f(x) = \frac{x+1}{x-1}$$

$$f(f(a)) = a \Rightarrow f(a) = \frac{a+1}{a-1} \Rightarrow f(f(a)) = \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = \frac{a+1+a-1}{a+1-a-1} = \frac{2a}{a-2} = a \checkmark$$

$$f(a) = \frac{a+1}{a-1} \rightarrow D_f: R - \{1\}$$

$$f(a) \neq 1 \Rightarrow \frac{a+1}{a-1} \neq 1 \Rightarrow a+1 \neq a-1 \Rightarrow 1 \neq -1 \text{ صحیح } \checkmark \Rightarrow R - \{1\}$$

$$f(x) = x^2 - x$$

$$g(x) = \sqrt{x^2 - 1} \xrightarrow{x \geq 1} (\sqrt{x^2 - 1})^2 - f(\sqrt{x^2 - 1}) = -x \Rightarrow x^2 - 1 - f(\sqrt{x^2 - 1}) = -x \Rightarrow \frac{x^2 - 1}{x^2} - \frac{f(\sqrt{x^2 - 1})}{x} + x = 0$$

$$f(g(a)) = -x$$

$$\Rightarrow x^2 - f(x) + x = 0 \Rightarrow (x-2)^2 = 0 \Rightarrow x=2 \Rightarrow \sqrt{x^2 - 1} = 2 \Rightarrow x^2 - 1 = 4 \Rightarrow x^2 = 5 \Rightarrow x = \pm \sqrt{5}$$

$$g(x) = \sqrt{x^2 - 1} \Rightarrow x^2 - 1 \geq 0 \Rightarrow x^2 \geq 1 \Rightarrow \begin{cases} x \geq 1 \\ x \leq -1 \end{cases} \Rightarrow \begin{cases} a \geq 1 \\ a \leq -1 \end{cases} \Rightarrow \sqrt{a^2 - 1} = \sqrt{5} = 2 \checkmark$$

$$f(x, y) = x^2 - 2xy + y^2$$

$$f(x) \times f(-x) \Rightarrow \begin{cases} x, y = x \Rightarrow x = 1 \Rightarrow 1 - 2 + 1 = 0 \Rightarrow f(x) = 0 \xrightarrow{x} \sqrt{x^2 - 2} = \sqrt{1 - 2} = \sqrt{-1} \\ x, y = -x \Rightarrow x = -1 \Rightarrow 1 - 2 + 1 = 0 \Rightarrow f(-x) = 0 \end{cases}$$

از این روش سنجش

$$f(\cos x + \sin x) = \tan x \xrightarrow{*} \frac{\sin x}{\cos x} = \frac{1 - \cos x}{\cos x}$$

$$f(x, a) = 3 \Rightarrow f \cos x + \sin x = 2/3 \Rightarrow \cos x + \sin x = 2/3 \Rightarrow \cos x + 1 - \cos^2 x = 2/3 \Rightarrow \cos^2 x - \cos x + 1/3 = 0$$

$$\cos^2 x + 1 - \cos^2 x - 2/3 = 0$$

$$\cos^2 x + 1 - \cos^2 x - 2/3 = 0$$

$$\cos^2 x + 1 - \cos^2 x - 2/3 = 0$$

$$\cos^2 x + 1 - \cos^2 x - 2/3 = 0$$

$$\cos^2 x + 1 - \cos^2 x - 2/3 = 0$$

$$\cos^2 x + 1 - \cos^2 x - 2/3 = 0$$

$$\cos^2 x + 1 - \cos^2 x - 2/3 = 0$$

$$\cos^2 x + 1 - \cos^2 x - 2/3 = 0$$

$$\cos^2 x + 1 - \cos^2 x - 2/3 = 0$$

$$\cos^2 x + 1 - \cos^2 x - 2/3 = 0$$

$$\textcircled{1} \rightarrow -1 \leq \cos x \leq 1 \Rightarrow \cos x = 1/2$$

$$\cos x = 1/2 \Rightarrow x = \pm \pi/3$$

$$\frac{1 - 1/2}{1/2} = \frac{1/2}{1/2} = 1 \textcircled{3}$$

①