

۲۰ آزمون عالی نوبت

نام و نام خانوادگی روزنامه تجاری پاسخنامه تشریحی تکلیف شماره کلاس پایه شماره
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الف) $f(x) = \sqrt{\frac{x-1}{x}} - \frac{2}{x-1} = \sqrt{\frac{x^2-1-2x}{x^2-x}} = \sqrt{\frac{1-2x}{x(x-1)}} = \frac{0}{\frac{1}{x} - \frac{1}{x-1}} = \frac{0}{\frac{1}{x} - \frac{1}{x-1}} = \frac{0}{\frac{x-1-x}{x(x-1)}} = \frac{0}{\frac{-1}{x(x-1)}} = 0 \cdot x(x-1) = 0$

ب) $f(x) = \frac{1}{x+1} - \frac{2}{x} = \frac{x - 2(x+1)}{x(x+1)} = \frac{x - 2x - 2}{x(x+1)} = \frac{-x - 2}{x(x+1)} = \frac{-x-2}{x(x+1)}$
 $\frac{1}{x-1} - \frac{1}{x+1} = \frac{x+1-x-1}{(x-1)(x+1)} = \frac{0}{(x-1)(x+1)} = 0$

① $x \neq 0, 1, -1, -2$

الف) $f(x) = \sqrt{(\frac{1}{x}-1)(x-1)} = \sqrt{(\frac{1-x}{x})(x-1)} = \sqrt{\frac{(1-x)(x-1)}{x}} = \sqrt{\frac{-(x-1)^2}{x}} = \frac{-(x-1)}{\sqrt{x}}$
 $D_f = (-\infty, -1] \cup [1, +\infty)$

ب) $\sqrt{x-1} + \sqrt{y+1} = 3$
 $\sqrt{y+1} = 3 - \sqrt{x-1}$
 $\sqrt{x-1} \geq 0 \Rightarrow 3 - \sqrt{x-1} \geq 0 \Rightarrow \sqrt{x-1} \leq 3 \Rightarrow x-1 \leq 9 \Rightarrow x \leq 10$
 $\sqrt{x-1} \geq 0 \Rightarrow \sqrt{x-1} \geq 0 \Rightarrow x-1 \geq 0 \Rightarrow x \geq 1$
 $\Rightarrow x \in [1, 10]$

$f(x) = \frac{\int_1^x (x^t - x - t)}{\sqrt{x^2-1} + 1} \rightarrow \frac{x^2 - x - 2}{(x-1)(x+1)} = \frac{-1}{x+1}$
 $\frac{x^2 - x - 2}{(x-1)(x+1)} = \frac{-1}{x+1} \Rightarrow x^2 - x - 2 = -x - 1 \Rightarrow x^2 - 1 = 0 \Rightarrow x = \pm 1$
 $D_f = (-\infty, -1) \cup (1, +\infty)$

$f(x) = \sqrt{3+ax-x^2} \Rightarrow \frac{-a}{-2} = \frac{b}{-2} \Rightarrow a = b$
 $x = -r \Rightarrow 3 - ra - f = 0 \Rightarrow 3 - ra - 1 = 0 \Rightarrow 2 = ra \Rightarrow a = \frac{2}{r}$
 $\Rightarrow \sqrt{3 + \frac{2}{r}x - x^2} = \frac{-x^2 - \frac{1}{r}x + 3}{\frac{2}{r}x + 3}$
 $E(x) = \sqrt{(x+r)(-x+\frac{r}{2})} \Rightarrow b = \frac{r}{2} \Rightarrow a+b = \frac{2}{r} + \frac{r}{2} = 1$

$f(x) = \begin{cases} 3x-2 & x \geq 1 \\ 2x+3 & x < 1 \end{cases}$

$g(x) = \sqrt{f(x) - x}$
 $x \geq 1 \Rightarrow \sqrt{3x-2-x} = \sqrt{2x-2} \Rightarrow 2x-2 \geq 0 \Rightarrow x \geq 1$
 $x < 1 \Rightarrow \sqrt{2x+3-x} = \sqrt{x+3} \Rightarrow x+3 \geq 0 \Rightarrow x \geq -3 \Rightarrow -3 \leq x < 1$
 $D_f = [-3, +\infty)$

$$f(x) = rx^r - f x$$

$$f\left(\sin \frac{\pi}{r}, \cos r\pi\right) \rightarrow \frac{\sqrt{r}}{r} + 1 = \frac{r + \sqrt{r}}{r}$$

$$\rightarrow \left(\frac{r + r + r\sqrt{r}}{r^2}\right) \times = r - r\sqrt{r} \Rightarrow \frac{r + r\sqrt{r} - 1 - r\sqrt{r}}{r} = \frac{-1}{r}$$

8

$$f(x) = \frac{x+1}{x-1}$$

$$f(a) = \frac{a+1}{a-1} \rightarrow f(f(a)) = \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = \frac{\frac{a+1+a-1}{a-1}}{\frac{a+1-a+1}{a-1}} \Rightarrow \frac{2a}{2} = a$$

$$\underline{a \in \mathbb{R} - \{1\}} \quad \checkmark \quad \leftarrow \text{Condition: } a \neq 1 \text{ ok}$$

9

$$f(x) = x^r - f x$$

$$g(x) = \sqrt{x^r - 1}$$

$$f(g(a)) = -f \rightarrow f(\sqrt{a^r - 1}) \Rightarrow a^r - 1 - f\sqrt{a^r - 1} + f = 0$$

$$\Leftrightarrow (\sqrt{a^r - 1} - r) \checkmark$$

$$a^r - 1 = f \rightarrow \frac{a^r - 1}{a} = \pm \sqrt{0}$$

10

$$f(x+r) = x^r - 0x + r$$

$$(x+r)^r \rightarrow x^r + rx^{r-1} + \dots + r = x - r \rightarrow (x+r)^r = (x+r)$$

$$\rightarrow f(f) \rightarrow 1r - f = 1r \quad \left\{ \begin{array}{l} 1r \times 1r = 1rf \\ f(1-r) \rightarrow 9 - (-r) = 1r \end{array} \right. \quad \checkmark$$

11

$$f(f \cos^2 x + 0 \cos x) = \tan^2 x$$

$$f(r, 0) \rightarrow f \cos^2 x + 0 \cos x = r, 0 \rightarrow \cos^2 x + 0 \cos x = \frac{r}{f}$$

$$(f \cos^2 x)(\cos^2 x + 0) = \left\{ \begin{array}{l} \cos x = \frac{r}{f} = \frac{+1}{f} \\ \cos x = \frac{-r}{f} \end{array} \right.$$

$$\rightarrow \tan^2 x = \frac{1 - \cos^2 x}{\cos^2 x} \Rightarrow \frac{1 - \frac{1}{f}}{\frac{1}{f}} = \frac{r}{f} \Rightarrow \frac{f-1}{f} = \frac{r}{f} \Rightarrow f-1=r$$

$$-1 \leq \cos x \leq 1 \Rightarrow \frac{r}{f} \in [-1, 1]$$

12