

19.5 آزمون

آزمون سنجش دانش و توانایی حل مسئله

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$$\text{الف) } \frac{x-1}{x} - \frac{x}{x-1} = 0 \rightarrow \frac{(x-1)^2 - x^2}{x(x-1)} = \frac{x^2 + 1 - 2x - x^2}{x(x-1)} = \frac{1-2x}{x(x-1)}$$

$x \neq 0, 1$

$$+ \frac{1}{x} - \frac{1}{x-1} = 0 \rightarrow \frac{x-1}{x(x-1)} - \frac{x}{x(x-1)} = 0 \rightarrow \frac{x-1-x}{x(x-1)} = 0 \rightarrow \frac{-1}{x(x-1)} = 0$$

$$\star D_f = (-\infty, 0) \cup (1, +\infty) \star$$

$$\text{ب) } \frac{1}{x+1} - \frac{1}{x} = 0 \rightarrow \frac{x - (x+1)}{x(x+1)} = 0 \rightarrow \frac{-1}{x(x+1)} = 0$$

$$\star D_f = \mathbb{R} \setminus \{-1, 0\} \star$$

$$\text{الف) } f(x) = \left(\frac{1}{x} \right)^2 - 4 \left(\frac{1}{x} - 2 \right) = \left(\frac{1}{x^2} - 4 \right) \left(\frac{1}{x} - 2 \right) = \left(\frac{1-x^2}{x^2} \right) \left(\frac{1-2x}{x} \right) = \frac{(1-x^2)(1-2x)}{x^3}$$

$$\star D_f = (-\infty, -2] \cup \left[\frac{1}{2}, +\infty \right) \star$$

$$\text{ب) } x-1 \geq 0 \rightarrow x \geq 1 \quad \sqrt{y+1} = 3 - \sqrt{x-1} \Rightarrow \sqrt{x-1} \leq 3 \Rightarrow x-1 \leq 9 \Rightarrow x \leq 10$$

$$\star D_f = [1, 10] \star$$

$$x^2 - x - 2 \geq 0 \quad (x-2)(x+1) \geq 0 \quad x \leq -1 \quad \text{یا} \quad x \geq 2 \quad x^2 - 1 \geq 0 \quad x \leq -1 \quad \text{یا} \quad x \geq 1$$

$$\star D_f = (-\infty, -1) \cup (2, +\infty) \star$$

$$3 + ax - x^2 = 0 \rightarrow x^2 - ax - 3 = 0 \quad \Delta = a^2 + 12 \geq 0 \quad (-2)b = -3 \rightarrow b = \frac{3}{2}$$

$$S_{x+y} = a_1 = a \quad -2 + \frac{3}{2} = a \rightarrow a = -\frac{1}{2}$$

$$\star \Rightarrow a+b = -\frac{1}{2} + \frac{3}{2} = 1 \star$$

$$f(x) - x \geq 0 \quad \text{① } 3x - 2 - x \geq 0 \quad 2x - 2 \geq 0 \rightarrow x \geq 1 \rightarrow [1, +\infty)$$

$$\text{② } 2x + 3 - x \geq 0 \rightarrow x + 3 \geq 0 \rightarrow x \geq -3 \rightarrow [-3, +\infty)$$

$$\star D_f = [-3, 1) \star$$

$$\sinh \frac{1}{x} + \cosh \frac{1}{x} = \sqrt{\frac{1}{x} + 1} \quad f\left(\sqrt{\frac{1}{x} + 1}\right) = 2\left(\sqrt{\frac{1}{x} + 1}\right)^2 - 4\left(\sqrt{\frac{1}{x} + 1}\right) = 2\left(\frac{1}{x} + 1\right) - 4\sqrt{\frac{1}{x} + 1} = \frac{2}{x} + 2 - 4\sqrt{\frac{1}{x} + 1}$$

$$\star D_f = \left(\frac{1}{4}, +\infty \right) \star$$

$$f(a) = \frac{a+1}{a-1} \quad f(f(a)) = \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = \frac{a+1+a-1}{a+1-a+1} = \frac{2a}{a+2}$$

$$f(f(a)) = \frac{2a}{a+2} \times \frac{a-1}{2} = a \quad \text{یعنی } a \neq 1 \quad \star a \in \mathbb{R} \setminus \{1\} \star$$

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$$g(a) = \sqrt{a^2 - 1} = t \rightarrow t^2 - a^2 t = -1 \Rightarrow t^2 - a^2 t + 1 = 0 \rightarrow (t - a)^2 = 0 \rightarrow t = a$$

$$g(a) = 1 \rightarrow \sqrt{a^2 - 1} = 1 \rightarrow a^2 - 1 = 1 \rightarrow a^2 = 2 \rightarrow a = \pm \sqrt{2} \star$$

$$x = 1 \rightarrow f(1) = 1 - \omega + 9 \Rightarrow f(1) = 1$$

$$\Rightarrow f(1) \times f(1) = 1 \times 1 = 1 \star$$

$$x = -9 \rightarrow f(-9) = 109 + 10 + 9 = 128$$

$$1 \cos^2 x + \omega \cos x = \frac{1}{p} \rightarrow 1 \cos^2 x + 10 \cos x = \frac{1}{p} \quad \cos x = t \rightarrow 1t^2 + 10t - \frac{1}{p} = 0$$

$$(1t - 1)(1t + 11) = 0 \rightarrow t = \frac{1}{p} \Rightarrow -\frac{1}{p} \quad |t| \rightarrow -1 \leq \cos x \leq 1$$

$$\tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{1 - \cos^2 x}{\cos^2 x}$$

$$= \frac{1 - \frac{1}{p^2}}{\frac{1}{p^2}} = p^2 - 1 \star \rightarrow f(p, \omega) = p^2 - 1 \star$$

$$\text{inf ٢) } \left(\left(\frac{1}{x} \right)^n - 9 \right) (3x - x^n) \geq 0 \quad \rightarrow \quad \begin{array}{c} -2 \quad \frac{5}{2} \\ + \quad | \quad - \quad | \quad + \\ \hline \end{array} \rightarrow Df = (-\infty, -2] \cup \left[\frac{5}{2}, +\infty \right)$$

$$\left(\frac{1}{x} \right)^n = 9 \rightarrow n = -2 \quad 3x = x^n \rightarrow n = \frac{5}{2}$$