

الف) $\frac{x-1}{x} - \frac{x}{x-1} = 0 \rightarrow \frac{(x-1)^2 - x^2}{x(x-1)} = \frac{x^2 + 1 - 2x - x^2}{x(x-1)} = \frac{1-2x}{x(x-1)}$

$x \neq 0, 1$

$D_f = (-\infty, 0) \cup [1, +\infty)$ ★

ب) $\frac{1}{x+1} - \frac{1}{x} = 0 \rightarrow \frac{x - (x+1)}{x(x+1)} = 0 \rightarrow \frac{-1}{x(x+1)} = 0$
 $\frac{1}{x-1} + \frac{1}{x+2} = 0 \rightarrow \frac{x+2 + x-1}{(x-1)(x+2)} = 0 \rightarrow \frac{2x+1}{(x-1)(x+2)} = 0$

$D_f = \mathbb{R} \setminus \{-\frac{1}{2}, -1, 0, 1\}$ ★

الف) $f(x) = \left(\frac{1}{x}\right)^2 - 4 = \frac{1}{x^2} - 4$

$\frac{1}{x^2} - 4 = 0 \rightarrow \frac{1}{x^2} = 4 \rightarrow x^2 = \frac{1}{4} \rightarrow x = \pm \frac{1}{2}$

★ $D_f = (-\infty, -\frac{1}{2}] \cup [\frac{1}{2}, +\infty)$ ★

ب) $x-1 \geq 0 \rightarrow x \geq 1$ $\sqrt{y+1} = 3 - \sqrt{x-1} \Rightarrow \sqrt{x-1} \leq 3 \Rightarrow x-1 \leq 9 \Rightarrow x \leq 10$

★ $D_f = [1, 10]$ ★

$x^2 - x - 2 \geq 0 \rightarrow (x-2)(x+1) \geq 0 \rightarrow x \leq -1 \text{ or } x \geq 2$

★ $D_f = (-\infty, -1) \cup (2, +\infty)$ ★

$3 + ax - x^2 = 0 \rightarrow x^2 - ax - 3 = 0$ $\Delta = a^2 + 12$ $(-2)b = -3 \rightarrow b = \frac{3}{2}$

$5x + 2 = a_1 = a$ $-2 + \frac{3}{2} = a \rightarrow a = -\frac{1}{2}$

★ $\Rightarrow a + b = -\frac{1}{2} + \frac{3}{2} = 1$ ★

$f(x) - x \geq 0 \rightarrow 3x - 2 - x \geq 0 \rightarrow 2x - 2 \geq 0 \rightarrow x \geq 1 \rightarrow [1, +\infty)$

$\sqrt{2x+3} - x \geq 0 \rightarrow \sqrt{2x+3} \geq x$
 If $x \geq 0$: $2x+3 \geq x^2 \rightarrow x^2 - 2x - 3 \leq 0 \rightarrow (x-3)(x+1) \leq 0 \rightarrow x \in [-1, 3]$
 If $x < 0$: $\sqrt{2x+3} \geq x$ is always true.

★ $D_f = [-1, 3]$ ★

$\sinh \frac{1}{x} + \cosh \frac{1}{x} = \sqrt{\frac{1}{x} + 1}$

$f(\sqrt{\frac{1}{x} + 1}) = 2(\sqrt{\frac{1}{x} + 1})^2 - 4(\sqrt{\frac{1}{x} + 1}) = 2(\frac{1}{x} + 1) - 4\sqrt{\frac{1}{x} + 1}$

$2(\frac{1}{x} + 1) - 4\sqrt{\frac{1}{x} + 1} = 0 \rightarrow \frac{1}{x} + 1 = 2\sqrt{\frac{1}{x} + 1} \rightarrow \sqrt{\frac{1}{x} + 1} = \frac{1}{2}$

$f(a) = \frac{a+1}{a-1}$

$f(f(a)) = \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = \frac{a+1+a-1}{a+1-a+1} = \frac{2a}{a+1}$

$f(f(a)) = \frac{2a}{a+1} \times \frac{a-1}{2} = a$

let $a \neq 1$

★ $a \in \mathbb{R} \setminus \{1\}$ ★

Subject:

Year:

Month:

Day:

$$g(a) = \sqrt{a^2 - 1} = t \rightarrow t^2 - at = -1 \Rightarrow t^2 - at + 1 = 0 \rightarrow (t - 1)^2 = 0 \rightarrow t = 1$$

$$g(a) = 1 \rightarrow \sqrt{a^2 - 1} = 1 \rightarrow a^2 - 1 = 1 \rightarrow a^2 = 2 \rightarrow a = \pm \sqrt{2} \quad \star$$

$$x = 1 \rightarrow f(1) = 1 - \omega + 9 \Rightarrow f(1) = 1 \Rightarrow f(1) \times f(1) = 1 \times 1 = 1 \quad \star$$

$$x = -9 \rightarrow f(-9) = 109 + 10 + 9 = 128$$

$$1 \cos^2 x + \omega \cos x = \frac{1}{p} \rightarrow 1 \cos^2 x + 10 \cos x = \frac{1}{p} \quad \cos x = t, \quad 1t^2 + 10t - \frac{1}{p} = 0$$

$$(1t - 1)(1t + 1) = 0 \rightarrow t = \frac{1}{p} \Rightarrow -\frac{1}{p} \quad |t| \rightarrow -1 \leq \cos x \leq 1 \quad \tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{1 - \cos^2 x}{\cos^2 x}$$

$$= \frac{1 - \frac{1}{p^2}}{\frac{1}{p^2}} = p^2 - 1 \rightarrow f(p, \omega) = p^2 - 1 \quad \star$$