

$$\text{الف)} \quad \frac{x-1}{x} - \frac{x}{x-1} \geq 0 \rightarrow \frac{(x-1)^2 - x^2}{x(x-1)} \geq 0 \rightarrow \frac{x^2 - 2x + 1 - x^2}{x(x-1)} \geq 0$$

$$\frac{-2x+1}{x(x-1)} \geq 0 \quad \frac{-\frac{1}{2} \quad 0 \quad 1}{+ \quad - \quad + \quad -} \quad D_f = (-\infty, -\frac{1}{2}] \cup (0, 1)$$

ب) $x \neq 1, x \neq -2, x \neq 0, x \neq -1$

$$\frac{x^2}{x-1} + \frac{1}{x+2} \neq 0$$

$$D = \mathbb{R} \setminus \left\{ -2, -1, -\frac{2}{3}, 0, 1 \right\} \quad x^2(x+2) + (x-1) = 0$$

$$x^3 + 2x^2 + x - 1 = 0$$

$$x + 0 = 0 \rightarrow x = -\frac{2}{3}$$

الف) $\left(\left(\frac{1}{x}\right)^x - 9\right)(x^2 - 9) \geq 0$

$\left(\frac{1}{x}\right)^x = 9 \quad \left(\frac{1}{x}\right)^x = x^x \rightarrow x^x = 9 \rightarrow x = \frac{9}{x} \rightarrow x^2 = 9 \rightarrow x = \pm 3$

$x = -2$

$$x \leq -2 \quad x \geq \frac{9}{x}$$

$$D_f = (-\infty, -2] \cup \left[\frac{9}{x}, +\infty\right)$$

ب) $x-1 \geq 0 \rightarrow x \geq 1$
 $y+1 \geq 0 \rightarrow y \geq -1$

$$\sqrt{y+1} = x - \sqrt{x-1} \rightarrow x - \sqrt{x-1} \geq 0 \rightarrow \sqrt{x-1} \leq x$$

$$x-1 \leq 1$$

$$x \leq 2$$

$$D_f = [1, 2]$$

الف) $x^2 - x - 2 \geq 0 \rightarrow (x-2)(x+1) \geq 0 \rightarrow x \leq -1, x \geq 2$

$x^2 - 1 \geq 0 \quad x \leq -1, x \geq 1, \sqrt{x^2-1} + 1 \geq 0$

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$$D_f = (-\infty, -1) \cup (2, \infty)$$

$-x^2 + ax + 3 \geq 0 \rightarrow$ چون $a \neq 0$ $\rightarrow$ $\Delta \geq 0, -2 \rightarrow 3 + (-2a) + 4 = 0$

$3a - 1 = 0 \rightarrow a = \frac{1}{3}$

$-x^2 - \frac{1}{3}x + 3 \rightarrow -(x+2)(x-\frac{3}{2}) \rightarrow \Delta = 2, \frac{3}{2} \rightarrow a+b = \frac{3}{2} - \frac{1}{2} = 1$

$x^2 + \frac{1}{3}x - 3$

$$f(x) - x \geq 0 \begin{cases} x \geq 1 \rightarrow x^2 - x - x \geq 0 \rightarrow x^2 - 2x \geq 0 \rightarrow x(x-2) \geq 0 \rightarrow x \geq 2 \rightarrow D_f = [2, +\infty) \\ x < 1 \rightarrow x^2 + x - x \geq 0 \rightarrow x^2 \geq 0 \rightarrow x \geq -1 \rightarrow D_f = [-1, 1) \end{cases}$$

$$D_g = [-1, 1) \cup [2, +\infty) \rightarrow D_g = [-1, +\infty)$$

$$f(x) = x^2 - x \rightarrow f\left(\frac{\sqrt{x}}{x} + 1\right) = \left(\frac{\sqrt{x}}{x} + 1\right)^2 = \frac{x}{x^2} + 1 + \sqrt{x} = \frac{1}{x} + 1 + \sqrt{x}$$

$$f\left(\frac{1}{x} + \sqrt{x}\right) - f\left(\frac{\sqrt{x}}{x} + 1\right) = \frac{1}{x} + 2\sqrt{x} - \sqrt{x} - 1 - \frac{1}{x} - 1 - \sqrt{x} = \frac{1}{x} - \frac{1}{x} - \frac{1}{x} = -\frac{1}{x} = \boxed{-\frac{1}{x}}$$

$$f\left(\frac{a+1}{a-1}\right) = a \rightarrow \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = a \rightarrow \frac{\frac{a+1+a-1}{a-1}}{\frac{a+1-a+1}{a-1}} = \frac{2a}{2} = a$$

برای $a \neq 1$

$$f(g(a)) = -f \rightarrow f(\sqrt{a^2-1}) = -f$$

$$(\sqrt{a^2-1})^2 - f\sqrt{a^2-1} = -f \rightarrow \sqrt{a^2-1} = t$$

$$t^2 - ft + f = 0 \rightarrow (t-f)^2 = 0 \rightarrow t = f$$

$$\sqrt{a^2-1} = f \rightarrow a^2-1 = f^2 \rightarrow \boxed{a = \pm \sqrt{f^2+1}}$$

$$x + x^2 = -x^2 \rightarrow x = -x \rightarrow f(-x) = x^2 + x^2 + x = 2x^2 + x$$

$$(x+x^2) = f \rightarrow x = 1 \rightarrow f(f) = 1 - 0 + f = f$$

$$f(f) \times f(-f) = 1 \times f$$

$$f \cos x + 2 \cos x = f + 2$$

$$\cos x (f \cos x + 2) = f + 2 \rightarrow \cos x = \frac{1}{f} \rightarrow \tan x = \left(\frac{\frac{\sqrt{x}}{x}}{\frac{1}{x}}\right)^2 = \boxed{x^2}$$