

الف)  $\frac{x-1}{x} - \frac{x}{x-1} \geq 0 \rightarrow \frac{(x-1)^2 - x^2}{x(x-1)} \geq 0 \rightarrow \frac{1-2x}{x(x-1)} \geq 0 \rightarrow \frac{1}{x} - \frac{2}{x-1} \geq 0$

$\rightarrow D_f = (-\infty, 0) \cup [\frac{1}{2}, 1)$

$\rightarrow x+1 \neq 0 \rightarrow x \neq -1, x \neq 0, x-1 \neq 0 \rightarrow x \neq 1, x+2 \neq 0 \rightarrow x \neq -2$

$\frac{2}{x-1} + \frac{1}{x+2} \neq 0 \rightarrow \frac{2(x+2) + (x-1)}{(x-1)(x+2)} \neq 0 \rightarrow \frac{3x+3}{(x-1)(x+2)} \neq 0$

$\rightarrow D_f = \mathbb{R} - \{-2, -\frac{3}{5}, -1, 0, 1\}$

الف)  $\left(\left(\frac{1}{3}\right)^x - 9\right)(32 - \varepsilon^x) \geq 0$   $\left(\frac{1}{3}\right)^x - 9 = 0 \rightarrow \left(\frac{1}{3}\right)^x = 3^2 \rightarrow x = -2$

$32 - \varepsilon^x = 0 \rightarrow \varepsilon^{2x} = \varepsilon^5 \rightarrow x = \frac{5}{2}$

$\rightarrow \frac{-2}{+} \frac{5}{-} \rightarrow D_f = (-\infty, -2] \cup [\frac{5}{2}, +\infty)$

ب)  $\sqrt[3]{x-1} + \sqrt{y+1} = 3 \rightarrow \sqrt{y+1} = 3 - \sqrt[3]{x-1}$

$x-1 \geq 0 \rightarrow x \geq 1, 3 - \sqrt[3]{x-1} \geq 0 \rightarrow \sqrt[3]{x-1} \leq 3 \rightarrow x-1 \leq 27 \rightarrow x \leq 28$

$\rightarrow D_f = [1, 28]$

$x^2 - x - 2 > 0 \rightarrow (x-2)(x+1) > 0 \rightarrow \frac{-1}{+} \frac{2}{-}$

$x^2 - 1 \geq 0 \rightarrow x^2 \geq 1 \rightarrow x \geq 1, x \leq -1$

$\sqrt{x^2-1} + 1 \neq 0$  همیشه مثبت

$D_f = (-\infty, -1) \cup (2, +\infty)$

$3 + ax - x^2 \geq 0$  if  $x=3$   $-\varepsilon + 3a + 3 = 0 \rightarrow a = \frac{1}{3}$

$-x^2 + \frac{x}{3} + 3 = 0 \xrightarrow{x^2} x^2 - x - 9 = 0 \rightarrow \begin{cases} x_1 = 2 \\ x_2 = b = -\frac{3}{2} \end{cases}$

$x^2 + x(-\frac{3}{2}) = 0 \rightarrow x = -\varepsilon, 3 \rightarrow (-\frac{3}{2})$

$a+b = -1$

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تابع درازنی تمام اعداد ماضی دینولتر از یک ناسقی است

if  $x \geq 1 \rightarrow g(x) = \sqrt{2x-2}$

تابع درازنی اعداد کچکتر از سقی سه منفی می شود

if  $x < 1 \rightarrow g(x) = \sqrt{x+3}$

$\rightarrow D_g = [-3, +\infty)$

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$$f\left(\underbrace{\sin \frac{\pi}{4}}_{=\frac{\sqrt{2}}{2}} + \underbrace{\cos 2\pi}_{=1}\right) = f\left(1 + \frac{\sqrt{2}}{2}\right) = 2\left(1 + \frac{\sqrt{2}}{2}\right)^2 - 4\left(1 + \frac{\sqrt{2}}{2}\right) = 2 + 2\sqrt{2} + \frac{2}{2} - 4 - 2\sqrt{2} = -1$$

$\rightarrow \boxed{-\frac{1}{2}}$

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درازنی تمامی مقادیر  $a$  تابع برقرار است فقط عرج نباید صفر باشد  $a \neq 1$

$$f(a) = \frac{a+1}{a-1} \quad f(f(a)) = \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = \frac{\frac{2a}{a-1}}{\frac{2}{a-1}} = a$$

$\rightarrow \boxed{a \in \mathbb{R} - \{1\}}$

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$$g(a) = \sqrt{a^2 - 1} = t \quad f(g(a)) = t^2 - \varepsilon t = -\varepsilon \rightarrow t^2 - \varepsilon t + \varepsilon = 0 \rightarrow (t-2)^2 = 0 \rightarrow t = 2$$

$$-t = 2 = \sqrt{a^2 - 1} \rightarrow a^2 - 1 = \varepsilon \rightarrow a^2 = \delta \rightarrow \boxed{a = \pm \sqrt{\delta}}$$

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$$f(x+3) = x^2 - 5x + 9 = (x-2)(x-3) \rightarrow f(x) = (x-5)(x-4)$$

$$\left. \begin{aligned} f(-3) &= (-1)(-9) = 9 \\ f(\varepsilon) &= (-1)(-2) = 2 \end{aligned} \right\} f(\varepsilon) \times f(-3) = 2 \times 9 = 18 \rightarrow \boxed{18\varepsilon}$$

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$$\varepsilon \cos^2 x + \delta \cos x = 3, \delta \rightarrow \varepsilon \cos^2 x + \delta \cos x - 3, \delta = 0 \rightarrow \cos x = \frac{-\delta \pm \sqrt{\delta^2 + 5\varepsilon}}{1}$$

$$\rightarrow \cos x = \frac{1}{4} \quad \left( x = \frac{\pi}{4} \right) \rightarrow \tan^2 \frac{\pi}{4} = 3$$

$\rightarrow \cos x = -\frac{3}{\varepsilon} \times \quad (-1 \leq \cos x \leq 1)$