

الف) $\frac{(x-1)^2 - x^2}{x(x-1)} \geq 0 \rightarrow \frac{1-x}{x(x-1)} \geq 0$ $\frac{0}{+\frac{1}{-} - \frac{1}{+} + \frac{1}{-}}$ $(-\infty, 0) \cup [4, 1)$

ب) $\frac{x-2x-2}{x(x+1)} \neq 0 \Rightarrow x \neq 0, 1, -\frac{1}{x}, 1, -2 \Rightarrow \mathbb{R} - \{-2, -\frac{1}{x}, -1, 0, 1\}$

الف) $\Rightarrow$ $\frac{-2}{+\frac{1}{-} - \frac{1}{+} + \frac{1}{-}}$ $(x^2 - 2)(x^2 - 1)$ $(-\infty, -2] \cup [\frac{1}{4}, +\infty)$

ب) $\sqrt{x-1} + \sqrt{y+1} = 3$
 $x-1 \geq 0$
 $x \geq 1$ ①
 $\sqrt{y+1} = 3 - \sqrt{x-1}$
 $y+1 \geq 0$
 $y \geq -1$ ②
 $\Rightarrow [1, 82]$

$\frac{x^2 - x - 2}{(x-2)(x+1)} > 0$ $\frac{-1}{+|-|+} \Rightarrow (-\infty, -1) \cup (2, +\infty)$ ①
 $x^2 - 1 \geq 0 \Rightarrow (x-1)(x+1) \geq 0 \Rightarrow (-\infty, -1] \cup [1, +\infty)$ ②
 $\Rightarrow (-\infty, -1) \cup (2, +\infty)$
 $\sqrt{x^2 - 1} \neq -1$
 همواره درست

$-x^2 + ax + 3 \geq 0 \Rightarrow -x^2 + (-1)x + 3 \geq 0 \Rightarrow x^2 - \frac{1}{x}x + 3 \geq 0$ $\frac{-2}{-\frac{1}{+} + \frac{1}{-} - \frac{1}{-}} \leq [-2, b]$
 $x - 2 \Rightarrow x^2 + x - 2 \leq 0 \Rightarrow x^2 + x - 12 \leq 0 \Rightarrow x = -2, \frac{12}{x} \Rightarrow b = \frac{12}{x}$
 $a + b = -\frac{1}{x} + \frac{12}{x} = 11$

$f(x) \geq x \Rightarrow$
 $x^2 - 2 \geq x \rightarrow x \geq 1 \rightarrow [1, +\infty)$ ①
 $x^2 + 3 \geq x, x < 1 \rightarrow x \geq 3, x < 1 \Rightarrow [-3, 1)$ ②
 $\Rightarrow ① \cup ② = [-3, +\infty)$

$$x = \sin \frac{\pi}{4} + \cos \frac{\pi}{4} \rightarrow \sin \frac{\pi}{4} + \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2} + 1 \rightarrow \frac{\sqrt{2}+1}{2} \Rightarrow f(x) = x^2 - 1 \Rightarrow y_2(x-1) \Rightarrow (1+\sqrt{2})x \frac{1+\sqrt{2}}{2} - 1$$

$$(1+\sqrt{2})\left(\frac{\sqrt{2}-1}{2}\right) = -\frac{1}{2}$$

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$$f(f(a)) = \frac{f(a)+1}{f(a)-1} = a \Rightarrow \frac{\frac{a+1}{a-1}+1}{\frac{a+1}{a-1}-1} = \frac{a+1+a-1}{a+1-a+1} = \frac{2a}{2} = a \Rightarrow \text{بالای تمامی نقاط } x \in \mathbb{R} - \{1\}$$

تابع متقابل معکوس است

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$$\left. \begin{array}{l} g(a) = \sqrt{a^2-1} \rightarrow f(\sqrt{a^2-1}) = -a \\ f(x) = x^2 - 1 \end{array} \right\} (\sqrt{a^2-1})^2 - 1 \sqrt{a^2-1} + a = 0 \Rightarrow (\sqrt{a^2-1} - 1)^2 = 0 \Rightarrow \sqrt{a^2-1} = 1 \Rightarrow a = \pm \sqrt{2}$$

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$$-r = x + r \rightarrow x = -r \rightarrow f(-r) = r^2 + r + r = 2r$$

$$r = x + r \rightarrow x = 1 \rightarrow f(1) = 1 - 1 + r = r$$

$$f(-r) \times f(r) = 2r \times r = 1 \times 1$$

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$$r \cos \frac{\pi}{4} + 1 \cos \frac{3\pi}{4} \rightarrow r \cos \frac{\pi}{4} + 1 \cos \frac{3\pi}{4} = 0 \xrightarrow{\times 2} 1 \cos \frac{\pi}{4} + 1 \cos \frac{3\pi}{4} = 0 \rightarrow \cos \frac{\pi}{4} + 1 \cos \frac{3\pi}{4} = 0$$

$$(\cos x + 1)(\cos x - 1) = 0$$

$$\cos x = \frac{1}{2} \text{ یا } \frac{-1}{2}$$

$$-1 \leq \cos x \leq 1 \quad \leftarrow \text{غیرقابل قبول} \quad \cos x = \frac{1}{2} \Rightarrow x = \frac{\pi}{3}$$

$$f(r/4) = \tan \frac{\pi}{4} = (\sqrt{r})^2 = r$$

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