

الف) $\frac{x-1}{x} - \frac{x}{x-1} \geq 0 \rightarrow \frac{(x-1)^2 - x^2}{x(x-1)} \geq 0$

$\frac{-2x+1}{x(x-1)} \geq 0$

Sign chart for $\frac{-2x+1}{x(x-1)}$:

$x < 0$	$0 < x < \frac{1}{2}$	$x = \frac{1}{2}$	$\frac{1}{2} < x < 1$	$x = 1$	$x > 1$
+	-	0	+	0	-

$D_f = (0, \frac{1}{2}) \cup (1, \infty)$

ب) $\frac{x+1}{x+2} \neq \frac{x}{x-1}$ $x \neq 0$ $x+2 \neq 0$ $x \neq -2$

$\frac{x+1}{x+2} - \frac{x}{x-1} \neq 0$

$\frac{(x+1)(x-1) - x(x+2)}{(x+2)(x-1)} \neq 0$

$\frac{x^2 - 1 - x^2 - 2x}{(x+2)(x-1)} \neq 0$

$\frac{-2x-1}{(x+2)(x-1)} \neq 0$

Sign chart for $\frac{-2x-1}{(x+2)(x-1)}$:

$x < -2$	$-2 < x < -\frac{1}{2}$	$x = -\frac{1}{2}$	$-\frac{1}{2} < x < 1$	$x = 1$	$x > 1$
+	-	0	+	0	-

$D_f = \mathbb{R} - \{-2, -\frac{1}{2}, 1\}$

الف) $(\frac{1}{x})^2 - 9 \leq (32 - x^2)$

$\frac{1}{x^2} - 9 \leq 32 - x^2$

$\frac{1}{x^2} + x^2 - 41 \leq 0$

$x^4 - 41x^2 + 1 \leq 0$

$x^2 = \frac{41 \pm \sqrt{1681 - 4}}{2} = \frac{41 \pm 41}{2}$

$x^2 = 41$ or $x^2 = 0$

$x = \pm \sqrt{41}$ or $x = 0$

Sign chart for $x^4 - 41x^2 + 1$:

$x < -\sqrt{41}$	$-\sqrt{41} < x < 0$	$0 < x < \sqrt{41}$	$x > \sqrt{41}$
+	-	-	+

$D_f = (-\infty, -\sqrt{41}) \cup (0, \sqrt{41})$

ب) $(\sqrt{y-1})^2 = (x-\sqrt{x-1})^2$

$x-1 \geq 0 \rightarrow x \geq 1$

$x - \sqrt{x-1} \geq 0 \rightarrow \sqrt{x-1} \leq x$

$\sqrt{x-1} \leq x \rightarrow x-1 \leq x^2 \rightarrow x^2 - x + 1 \geq 0$

$x^2 - x + 1 \geq 0 \rightarrow (x-\frac{1}{2})^2 + \frac{3}{4} \geq 0$

$x \in \mathbb{R}$

Sign chart for $x^2 - x + 1$:

$x < -1$	$-1 < x < 2$	$x > 2$
+	-	+

$D_f = (-\infty, -1) \cup (2, \infty)$

الف) $\sqrt{-x^2+4x+3}$

$-x^2+4x+3 \geq 0$

$x^2 - 4x - 3 \leq 0$

$x = \frac{4 \pm \sqrt{16+12}}{2} = \frac{4 \pm 5}{2}$

$x = \frac{9}{2}$ or $x = -\frac{1}{2}$

Sign chart for $x^2 - 4x - 3$:

$x < -\frac{1}{2}$	$-\frac{1}{2} < x < \frac{9}{2}$	$x > \frac{9}{2}$
+	-	+

$D_f = [-\frac{1}{2}, \frac{9}{2}]$

ب) $a+b = \frac{1}{x} + \frac{1}{y} = 1$

$\frac{1}{x} + \frac{1}{y} = 1 \rightarrow \frac{x+y}{xy} = 1 \rightarrow x+y = xy$

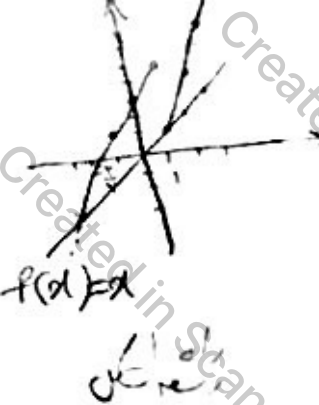
$xy - x - y = 0 \rightarrow (x-1)(y-1) = 1$

$x-1 = t \rightarrow y-1 = \frac{1}{t} \rightarrow y = \frac{1+t}{t}$

$x = 1+t$

$t \neq 0$

$D_f = \mathbb{R} - \{0\}$



x	1	$\sqrt{3}$	2
$f(x)$	1	2	3

x	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$f(x)$	1	2	3

$f(x) = x$
 $f(x) > x$
 $D_f = [-2, +\infty)$

$f(x) = 2\left(\frac{\sqrt{3}+1}{2}\right)^2 - 2\left(\frac{\sqrt{3}+1}{2}\right) = \frac{2(\sqrt{3}+1)^2}{4} - \frac{2(\sqrt{3}+1)}{2}$
 $\sin\left(\frac{\pi}{6} + \cos\right)x = \frac{\sqrt{3}+1}{2}$
 $(x) \in \dots$

$f(a) = \frac{a+1}{a-1}$
 $f\left(\frac{a+1}{a-1}\right) = \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = \frac{a+1+a-1}{a+1-a+1} = \frac{2a}{2} = a$

$D_f = \{a \mid a \in \mathbb{Z}, a \neq 1\}$

$g(a) = \sqrt{a^2-1}$
 $f(\sqrt{a^2-1}) = \frac{\sqrt{a^2-1}}{a^2-1} = \frac{1}{\sqrt{a^2-1}}$

$\rightarrow (-\sqrt{a^2-1})^2 = \frac{a^2-1}{-a^2-1} \rightarrow 1(a^2-1) = -a^2-1$
 $\rightarrow a^2 - 1 - a^2 + 1 = 0 \rightarrow 0 = 0 \rightarrow a = \pm\sqrt{0}$

$f(-2) = (2-2)(2-2) = (4-2)(-4-2) = -2(-6) = 12$
 $a+2=-2$
 $a=-4$

$f(2) = (2-2)(2-2) = (1-2)(1-2) = -2(-1) = 2$
 $x+2=2$
 $x=0$

$\cos x + 0 \cos x = 0 \rightarrow \cos x + 0 \cos x - 0 = 0$
 $\cos x + 0 \cos x - 0 = 0$
 $(\cos x + 1)(\cos x - 1) \rightarrow \cos x = \frac{1}{2} \rightarrow x = \frac{\pi}{3} + 2k\pi$