

۱- $f(x) = \sqrt{\frac{x-1}{x} - \frac{1}{x-1}} = \sqrt{\frac{(x-1)^2 - x^2}{x(x-1)}} = \sqrt{\frac{1-2x}{x(x-1)}} \rightarrow \frac{1}{2}$

۲- $f(x) = \frac{1}{x+1} - \frac{1}{x} = -\frac{1}{x(x+1)} \rightarrow 0$

مخرج و صورت کثیر و جود در مخرج حل
باید منفی باشد

$D_f = (-\infty, 0) \cup [\frac{1}{2}, +\infty)$

$D_f = \mathbb{R} - \{ \frac{1}{2}, -\frac{1}{2}, -1, 0 \}$

۳- $f(x) = \sqrt{(\frac{1}{x}-1)(3x-4)}$

بعضی از عبارات
نامعنی باید باشد

$x = -\frac{1}{2}$ $x = \frac{4}{3}$

$D_f = (-\infty, -\frac{1}{2}] \cup [\frac{4}{3}, +\infty)$

۴- $\sqrt{x-1} + \sqrt{y+1} = 3 \Rightarrow \sqrt{x-1} - 3 = -\sqrt{y+1} \Rightarrow \sqrt{x-1} - 3 \leq 0$

منفی باشد

$\Rightarrow \sqrt{x-1} < 3 \Rightarrow x-1 < 9 \Rightarrow x < 10$

$D_f = \mathbb{Q} \cap \mathbb{Q} = [1, 10]$

۵- $f(x) = \frac{\log(x^2-x-2)}{\sqrt{x^2-1}+1}$

$x^2-x-2 > 0 \Rightarrow (x-2)(x+1) > 0$

$x^2-1 > 0 \Rightarrow (x-1)(x+1) > 0$

$D_f = \mathbb{Q} \cap \mathbb{Q} = (-\infty, -1) \cup (2, +\infty)$

۶- $f(x) = \sqrt{3+ax-x^2}$ $D_f = [-2, b]$

$a+b = ? \Rightarrow -\frac{1}{2} + \frac{3}{2} = \frac{1}{2} = 1$

$-2 \Rightarrow \sqrt{3-2a-4} = 0 \Rightarrow \sqrt{-2a-1} = 0 \Rightarrow a = -\frac{1}{2}$

$\sqrt{-2a-1} = 0 \Rightarrow -2a-1 = 0 \Rightarrow a = -\frac{1}{2}$

$b = \frac{3}{2}$

۷- $f(x) = \begin{cases} 3x-2 & x \geq 1 \\ 3x+3 & x < 1 \end{cases}$ $g(x) = \sqrt{f(x)-x} \Rightarrow f(x) - x \geq 0$

$\Rightarrow f(x) \geq x$

if $x \geq 1 \Rightarrow f(x) \geq x \Rightarrow 3x-2 \geq x \Rightarrow x \geq 1 \Rightarrow [1, +\infty)$

if $x < 1 \Rightarrow f(x) \geq x \Rightarrow 3x+3 \geq x \Rightarrow x \geq -3 \Rightarrow [-3, 1)$

$D_f = [-3, +\infty)$

$f(x) = 2x^2 - 5x$, $f\left(\sin \frac{\pi}{4} + \cos 2\pi\right) = ?$ - 9
 $\frac{\sqrt{2}}{2} + 1 = 2 + \frac{\sqrt{2}}{2}$ $\frac{2(2+\sqrt{2})^2}{4} - 5(2+\frac{\sqrt{2}}{2})$
 $\Rightarrow \frac{2 + \frac{\sqrt{2}}{2} - 1 - \frac{5\sqrt{2}}{2}}{2} = \boxed{-\frac{1}{2}}$ $\frac{2 + \frac{\sqrt{2}}{2}}{2} = 1 + \frac{\sqrt{2}}{4}$

$f(x) = \frac{x+1}{x-1}$, $f(f(x)) = x$, $x = ?$ - 7
 $f(x) = \frac{x+1}{x-1} \Rightarrow f(f(x)) = \frac{\frac{x+1}{x-1} + 1}{\frac{x+1}{x-1} - 1} = \frac{x+1+x-1}{x-1-x+1} = \frac{2x}{-x+2} = x$
 $\Rightarrow \frac{2x}{-x+2} = x \Rightarrow 2x = -x^2 + 2x \Rightarrow x^2 = 0 \Rightarrow x = 0$
به ازای $x=0$ برقرار نیست (تقریباً)
در دامنه برقرار است.
گداز دامن حذف کرده ایم.

$f(x) = x^2 - 5x$, $g(x) = \sqrt{x^2 - 1}$, $f(g(x)) = -5$ - 1
 $x^2 - 5x = -5 \Rightarrow x^2 - 5x + 5 = 0 \Rightarrow (x-2)^2 = 0 \Rightarrow x = 2 \Rightarrow g(x) = 2$
 $\sqrt{x^2 - 1} = 2 \Rightarrow x^2 - 1 = 4 \Rightarrow x^2 = 5 \Rightarrow x = \pm \sqrt{5}$
 $x^2 > 1 \Rightarrow x > 1$
 $\Rightarrow x = \sqrt{5}$

$f(x+y) = x^2 - 2x + y$, $f(x) \times f(-5) = ?$ - 9
 $x+5 = 5 \Rightarrow x = 0 \Rightarrow 1 - 10 + 5 = -4$
 $x+5 = -5 \Rightarrow x = -10 \Rightarrow (-10)^2 - 2(-10) + 5 = 115$

$f(\cos^2 x + 2 \cos x) = \tan^2 x$, $f(1/5) = ?$ - 10
 $f(\cos^2 x + 2 \cos x) = \frac{1}{5} \Rightarrow \cos^2 x + 2 \cos x - \frac{1}{5} = 0$
 $\Rightarrow (2 \cos x + \frac{1}{5})(2 \cos x - \frac{4}{5}) = 0$
 $\cos x = \frac{1}{5}$
 $-1 < \cos x < 1$
 $\tan^2 x + 1 = \frac{1}{\cos^2 x} \Rightarrow \tan^2 x + 1 = 25 \Rightarrow \tan^2 x = 24$
 $\tan x = \pm \sqrt{24}$