

پ. انجمن عالی نوشتن

$$f(n) = \sqrt{\frac{n-1}{n} - \frac{n}{n-1}} = \sqrt{\frac{(n-1)^2 - n^2}{n^2 - n}} = \sqrt{\frac{1-2n}{n^2-n}} \rightarrow \frac{1-2n}{n^2-n} \geq 0 \rightarrow \frac{1}{n} - \frac{2}{n-1} + \frac{1}{n} \geq 0$$

$$\rightarrow D_f = (-\infty, 0) \cup \left(\frac{1}{2}, 1\right)$$

$$\rightarrow f(n) = \frac{\frac{1}{n-1} - \frac{n}{n}}{\frac{n}{n-1} + \frac{1}{n+2}} \rightarrow \begin{cases} n \neq -1, n \neq 0 \\ n \neq 1, n \neq -1 \end{cases}$$

$$\rightarrow \frac{\frac{n-1}{n-1} + \frac{1}{n+2}}{\frac{n-1}{n-1} + \frac{1}{n+2}} \neq 0 \rightarrow \frac{n+4+n-1}{(n-1)(n-2)} \neq 0 \rightarrow \frac{2n+3}{(n-1)(n-2)} \neq 0 \rightarrow n \neq -\frac{3}{2}$$

$$f(n) = \sqrt{\left(\left(\frac{1}{n}\right)^n - 9\right)(32 - 2^n)} \rightarrow \left(\left(\frac{1}{n}\right)^n - 9\right)(32 - 2^n) \geq 0 \rightarrow (n^{-n} - 9)(2^5 - 2^n) \geq 0$$

$$\rightarrow \frac{-x}{+1} - \frac{9}{-1} + \rightarrow D_f = (-\infty, -2] \cup \left[\frac{5}{2}, +\infty\right)$$

$$\rightarrow \sqrt{x-1} + \sqrt{y+1} = 3 \rightarrow x-1 \geq 0 \rightarrow x \geq 1$$

$$\rightarrow D_f = [1, 12]$$

$$f(n) = \frac{\log_{\frac{1}{e}}(n^2 - n - 2)}{\sqrt{n^2 - 1} + 1} \rightarrow \begin{cases} n^2 - n - 2 > 0 \rightarrow (n-2)(n+1) > 0 \rightarrow \frac{-1}{+1} - \frac{2}{-1} + \rightarrow (-\infty, -1) \cup (2, +\infty) \\ n^2 - 1 \geq 0 \rightarrow n \geq 1, n \leq -1 \end{cases}$$

$$\rightarrow D_f = (-\infty, -1) \cup (2, +\infty)$$

$$f(n) = \sqrt{3+an-n^2} \xrightarrow{n=-2} 3-2a-2=0 \rightarrow a = -\frac{1}{2}$$

$$\rightarrow f(n) = \sqrt{-n^2 - \frac{1}{2}n + 3} \rightarrow -n^2 - \frac{1}{2}n + 3 \geq 0 \rightarrow -(n+2)(n-\frac{3}{2}) \geq 0 \rightarrow D_f = [-2, \frac{3}{2}] \rightarrow b = \frac{3}{2}$$

$$\rightarrow a+b = -\frac{1}{2} + \frac{3}{2} = 1$$

$$f(n) = n \geq 0 \rightarrow f(n) \geq n \rightarrow \begin{cases} 4n-2 \geq n \rightarrow 3n \geq 2 \rightarrow n \geq \frac{2}{3} \\ 2n+3 \geq n \rightarrow n \geq -3 \rightarrow -3 \leq n < 1 \end{cases}$$

$$\rightarrow D_g = [-3, +\infty)$$

$f\left(\sin \frac{\pi}{\sqrt{r}} + \cos \sqrt{r}\pi\right) = f\left(\sin 40^\circ + \cos 0^\circ\right) = f\left(\frac{\sqrt{r}}{r} + 1\right) = f\left(r + \frac{\sqrt{r}}{r}\right)$ $\rightarrow f(n) = r \times (n - r)$ $\rightarrow f\left(r + \frac{\sqrt{r}}{r}\right) = r + \sqrt{r}\left(\frac{\sqrt{r}}{r} - 1\right) \rightarrow \sqrt{r} + \frac{r}{r} - r - \sqrt{r} = \boxed{\frac{-1}{r}}$	٤
$f(a) = \frac{a+1}{a-1} \rightarrow f(f(a)) = \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = \frac{\frac{ra}{a-1}}{\frac{r}{a-1}} = a$ $\rightarrow \boxed{a \in \mathbb{R} - \{1\}}$	٧
$f(x) = -2 \rightarrow x^r - 2x = -2 \rightarrow x^r - 2x + 2 = 0 \rightarrow (x-2)^r = 0 \rightarrow x = 2$ $\rightarrow g(a) = r \rightarrow \sqrt{a^r - 1} = r \rightarrow a^r - 1 = 2 \rightarrow \boxed{a = \pm \sqrt[r]{3}}$	٨
$n = 1 \rightarrow f(2) = 1 - 2 + 4 = 3$ $n = -4 \rightarrow f(-3) = 3 \times 4 + 3 \times 0 + 4 = 16$ $f(2) \times f(-3) = 3 \times 16 = \boxed{48}$	٩
$2 \cos^r n + 2 \cos n = \sqrt{13} \rightarrow 2 \cos^r n + 2 \cos n - \frac{\sqrt{r}}{r} = 0 \rightarrow \cos^r n + 2 \cos n - 12 = 0$ $\rightarrow \left(\cos n + \frac{\sqrt{r}}{2}\right) \left(\cos n - \frac{r}{2}\right) = 0 \rightarrow \cos n = -\frac{\sqrt{r}}{2} \rightarrow \text{غير ممكن قبل}$ $\cos n = \frac{1}{r} \checkmark$ $\frac{1}{\cos^r n} = 1 + \tan^r n \rightarrow \tan^r n = \sqrt[r]{2} - 1 = \boxed{3}$	١٠