

۲. آنالیز علی نوری

الف) $\sqrt{\frac{x-1}{x}} - \frac{x}{x-1}$ $\frac{x-1}{x} - \frac{x}{x-1} \geq 0$ $\frac{(x-1)^2 - x^2}{x(x-1)} \geq 0$ $\frac{x^2 - 2x + 1 - x^2}{x(x-1)} \geq 0$ $\frac{-2x + 1}{x(x-1)} \geq 0$ $x = \frac{1}{2}$ $x = 0$ $x = 1$ $Df = (-\infty, 0) \cup [\frac{1}{2}, 1)$	ب) $\frac{1}{x+1} - \frac{x}{x}$ $\frac{1}{x+1} + \frac{1}{x+2}$ $\frac{x}{x-1} + \frac{1}{x+2} \neq 0$ $\frac{x^2 + 4x + x - 1}{(x-1)(x+2)} \neq 0$ $\frac{x^2 + 5x - 1}{(x-1)(x+2)} \neq 0$ $x \neq -1$ $x \neq 0$ $x \neq 1$ $x \neq -2$ $D = R - \{-2, -\frac{1}{5}, -1, 0, 1\}$ $x \neq -\frac{1}{5}$ $x \neq -5$
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الف) $\sqrt{(\frac{1}{x})^2 - 9} (x^2 - 4) \geq 0$ $\frac{1}{x^2} = 9 \rightarrow x = -\frac{1}{3}$ $x^2 = 4 \rightarrow x = \pm 2$ $\rightarrow x = \frac{1}{3}$ $Df = R - (-2, \frac{1}{3})$ $\frac{1}{2} (-\infty, -2] \cup [\frac{1}{3}, +\infty)$	ب) $\sqrt{x-1} + \sqrt{y+1} = 3$ $x-1 \geq 0$ $y+1 \geq 0$ $x \geq 1$ $y \geq -1 \rightarrow y \in [-1, +\infty)$ $x \in [1, +\infty)$ $\sqrt{y+1} = 3 - \sqrt{x-1}$ $\sqrt{y+1} \leq 3$ $x-1 \leq 1$ $Df = [1, 2]$
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$(x^2 - x - 2) \log x \pm 1 \geq 0$ $\sqrt{x^2 - 1} + 1$ $x^2 - 1 \geq 0$ $x^2 \geq 1$ $x \geq 1$ $x \leq -1$ $Df = (-\infty, -1) \cup (2, +\infty)$	$x^2 - x - 2 \geq 0$ $(x-2)(x+1)$ $x = 2$ $x = -1$ -1 2 $+ \quad - \quad +$ -1 0 1 2
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$f(x) = \sqrt{ax - x^2 + 3} \rightarrow \frac{1}{x} x - x^2 + 3 \geq 0$ $ax + 3 - x^2 = 0$ $x = -1$ $-2a - 4 + 3 = 0$ $-2a = 1$ $a = -\frac{1}{2}$ $\frac{1}{x} - 2 = 0$ $x = \frac{1}{2}$ $x = \frac{1}{x} \pm \frac{1}{x} = -2 \rightarrow \frac{1}{x} \rightarrow b$ $\frac{1}{x} - 2(-3) = 0$ $\frac{1}{x} = 6$ $x = \frac{1}{6}$ $-\frac{1}{x} + \frac{1}{x} = 0 = a + b$	$x = \frac{1}{x} \pm \frac{1}{x} = -2 \rightarrow \frac{1}{x} \rightarrow b$
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$f(x) \geq x \rightarrow D = x \geq 1 \cup x \geq -3$ $x = 1 \rightarrow f(x) = x$ $x = 0 \rightarrow f(x) = 3$ $x = 2 \rightarrow f(x) = 2$ $x = -3 \rightarrow f(x) = x$ $Df = [-3, +\infty)$	$x = 1$ $x = 2$ $x = -3$
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$$r\left(\frac{\sqrt{r}}{r} + 1\right)^r - r\left(\frac{\sqrt{r}}{r} + 1\right)$$

$$\rightarrow r\left(\frac{r}{r} + 1 + \sqrt{r}\right) - r\sqrt{r} - r$$

$$\frac{r}{r} + \frac{r}{r} + \sqrt{r} - r\sqrt{r} = r - r$$

$$\frac{r}{r} - \frac{r}{r} = \boxed{-\sqrt{r}} \quad \text{D}$$

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$$f(a) = \frac{a+1}{a-1} \quad \frac{a+1+a-1}{a-1} = a \times \frac{a+1-a+1}{a-1}$$

$$\frac{a+1}{a-1} + 1 = a \quad \frac{ra}{a-1} = a \times \frac{r}{a-1}$$

$$\frac{a+1}{a-1} - 1 \rightarrow a \neq 1$$

$$\boxed{a \in \mathbb{R} - \{1\}} \quad \text{D}$$

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$$g(a) = \sqrt{a^2 - 1}$$

$$f(x) \rightarrow a^2 - 1 - f(\sqrt{a^2 - 1}) = -r$$

$$a^2 - 1 - \sqrt{a^2 - 1} = -r \rightarrow (\sqrt{a^2 - 1} - r)^2 = 0$$

$$a^2 - 1 - \sqrt{a^2 - 1} + r = 0 \quad \sqrt{a^2 - 1} = r$$

$$\sqrt{a^2 - 1} = \sqrt{r}$$

$$a^2 - 1 = r$$

$$a^2 = r + 1$$

$$a = \pm \sqrt{r + 1} \quad \text{D}$$

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$$f(x+r) = x^2 - ax + a$$

$$f(x+r) = (x-r)(x-r)$$

$$x+r = -r$$

$$x = -r$$

$$f(r) = 1 - a + a = r$$

$$f(-r) = r^2 + r^2 + a \rightarrow r^2$$

$$x = \boxed{1 \pm r} \quad \text{D}$$

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$$f(\cos^2 x + a \cos x) = \tan^2 x \quad \frac{\sin^2 x}{\cos^2 x}$$

$$r + r + a^2 = r^2$$

$$x^2 \rightarrow 1 + r + 10t = r$$

$$1 + r + 10t - r = 0$$

$$a = 100 - 2(-29) = 138$$

$$\boxed{f(138) = r} \quad \text{D}$$

$$1 \leq \cos \leq 1$$

$$\cos = \frac{1}{r} \quad \sin = \frac{\sqrt{r^2 - 1}}{r}$$

$$\tan = \frac{\sqrt{r^2 - 1}}{1}$$

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