

في مجال حقيقي في مجال مركب في مجال حقيقي

1-

$$a) f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}} \rightsquigarrow \sqrt{\frac{(x-1)^2 - x^2}{x(x-1)}} \rightarrow \sqrt{\frac{x^2 + 1 - 2x - x^2}{x(x-1)}}$$

$$-x+1=0 \Rightarrow x=1 \quad \frac{-x+1}{x(x-1)} \geq 0 \rightsquigarrow \frac{0}{+} - \frac{1}{+} + \frac{1}{+} \quad D_f = (-\infty, 0) \cup [\frac{1}{2}, 1)$$

$$b) \frac{x-1}{x+1} - \frac{x}{x-1} \rightarrow x \neq 0 \quad \frac{1}{x-1} + \frac{1}{x+1} \rightarrow \frac{x^2 + 1 + x - 1}{(x-1)(x+1)} \quad f(x) + 0 \rightarrow x \neq -\frac{1}{2}$$

$$x \neq 1 \quad \frac{x}{x-1} + \frac{1}{x+1} \rightarrow x \neq -1 \quad D_f = \mathbb{R} \setminus \{-1, -\frac{1}{2}, 1, 0, 1\}$$

$$c) f(x) = \sqrt{\left(\left(\frac{1}{x}\right)^x - 9\right)(x^x - 9)} \quad \left(\left(\frac{1}{x}\right)^x - 9\right)(x^x - 9) \geq 0$$

$$\left(\left(\frac{1}{x}\right)^x - 9\right) \geq 0 \rightarrow \left(\frac{1}{x}\right)^x \geq 9 \rightsquigarrow x \leq -1 \quad / \quad x^x - 9 \geq 0 \rightarrow x^x \geq 9$$

$$x^x \geq 9 \rightarrow x > 2 \quad \text{or} \quad x < -1 \quad \text{or} \quad x > 2 \quad \text{or} \quad x < -1$$

$$\text{if } x < -1 \rightarrow \left(\left(\frac{1}{x}\right)^x - 9\right)(x^x - 9) \rightarrow \text{جواب} \quad D_f = (-\infty, -1] \cup [2, +\infty)$$

$$d) \sqrt{x-1} + \sqrt{y+1} = x \rightsquigarrow \sqrt{y+1} = x - \sqrt{x-1} \rightarrow x - \sqrt{x-1} \geq 0$$

$$x \geq 1 \quad x \geq \sqrt{x-1} \rightarrow x^2 \geq x-1 \rightarrow x^2 - x + 1 \geq 0 \quad D_f = [1, +\infty)$$

$$e) f(x) = \frac{\log x}{\sqrt{x^2-1} + 1} \rightsquigarrow (x^2 - x - 2) \rightsquigarrow (x-2)(x+1) > 0 \quad \frac{-1}{\frac{1}{2} - \frac{1}{2} + 1}$$

$$x^2 - 1 \geq 0 \quad \sqrt{x^2-1} + 1 \quad D_f = (-\infty, -1) \cup (1, +\infty)$$

$$f) f(x) = \sqrt{x^2 + ax - x^2} \rightsquigarrow D_f = [-r, b]$$

$$\text{MR NOTE: } a+b \geq \frac{r}{r} - \frac{1}{r} \geq \frac{r}{r} \geq 1 \quad \text{or} \quad -r + (-ra) + r = 0 \quad -ra \geq 1 \rightarrow a \leq -\frac{1}{r}$$



3- $f(x) = \begin{cases} x^2 - x & x > 1 \\ x + 1 & x < 1 \end{cases}$ $g(x) = \sqrt{f(x) - x} \rightarrow f(x) - x > 0$

1) $x^2 - x - x > 0 \rightarrow x^2 - 2x > 0 \rightarrow x(x-2) > 0 \rightarrow x < 0 \vee x > 2$
 $x + 1 - x > 0 \rightarrow 1 > 0$ always true.

Domain: $x < 0 \vee x > 2$ or $x < 1$.
 $-3 \leq x < 1$ and $x > 2$.
 $\text{Domain} = [-3, 1) \cup (2, \infty)$

4- $f(x) = x^2 - tx$ $f(\sin \frac{\sqrt{r}}{r} + \cos \frac{\sqrt{r}}{r})$

$\hookrightarrow r(1 + \sqrt{\frac{r}{r}})^2 - t(1 + \sqrt{\frac{r}{r}}) = r + \frac{r}{r} + r\sqrt{\frac{r}{r}} - t - r\sqrt{\frac{r}{r}} = \frac{-t}{r} + \frac{r}{r} = \frac{-1}{r}$

$\hookrightarrow (1 + \frac{r}{r} + \sqrt{r})r - t - r\sqrt{r}$

5- $f(x) = \frac{x+1}{x-1}$ $f(f(a)) = a \rightarrow \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = a$

$\text{Domain: } \mathbb{R} - \{1\}$

$\hookrightarrow \frac{a+1}{a-1}$

$\frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = a \rightarrow \frac{a+1}{a-1} - 1 = \frac{r}{a-1} \rightarrow a \neq 1$

6- $f(x) = x^2 - tx$, $g(x) = \sqrt{x^2 - 1} \rightarrow f(g(a)) = -t$

$t^2 - ft = -t \rightarrow t^2 - ft + t = 0 \rightarrow t(t - f + 1) = 0$

$\hookrightarrow g(a) = t$

$(t - r)^2 = 0 \rightarrow t = r \rightarrow \sqrt{a^2 - 1} = r \rightarrow a^2 - 1 = r^2 \rightarrow a^2 = r^2 + 1 \rightarrow a = \pm \sqrt{r^2 + 1}$

7- $f(x+y) = x^2 - ax + 4$

$x+y = 1 \rightarrow x=1 \rightarrow 1 - a + 4 = 1 \rightarrow a = 4$

$x+y = -1 \rightarrow x=-1 \rightarrow 1 - a + 4 = -1 \rightarrow a = 6$

$f(1) \times f(-1) = 1 \times 1 = 1$

10- $f(\cos^2 x + \sin^2 x) = \tan^2 x$ $f(\frac{r}{\omega})$

$f(\cos^2 x + \sin^2 x) = \tan^2 x \rightarrow \cos^2 x = t \rightarrow \tan^2 x = \frac{1-t}{t}$

$t = \frac{-10 \pm 14}{14}$, $\frac{-4}{14} = t$

$\Delta = 100 + 144 = 244 \rightarrow \sqrt{244} = 14$

$\hookrightarrow -1 \leq \cos x \leq 1 \rightarrow \cos x = \frac{1}{r}$

$\tan^2 x = \frac{r^2 - 1}{r^2}$