

۲. آنالیز عادی نوشتن

الف) $\frac{(n-1)^2 - x^2}{x(n-1)} \geq 0 \rightarrow \frac{x^2 + 1 - 2x - x^2}{x(n-1)} \geq 0 \rightarrow \frac{1-2x}{x(n-1)} \geq 0 \rightarrow \begin{cases} x=0 \\ x=1 \\ x=\frac{1}{2} \end{cases}$

ب) $\frac{x-2x-2}{x(n+1)} = \frac{-(n+2)}{x(n+1)} \rightarrow \begin{cases} x=-2 \\ x=0 \\ x=-1 \end{cases}$
 صورت کسر: $\frac{x-2x-2}{x(n+1)} = \frac{-(n+2)}{x(n+1)} \rightarrow \begin{cases} x=-2 \\ x=0 \\ x=-1 \end{cases}$
 مخرج کسر: $\frac{x^2+2x-1}{(x-1)(n+2)} = \frac{x^2+2x-1}{(x-1)(n+2)} \rightarrow \begin{cases} x=-\frac{1}{2} \\ x=1 \\ x=-2 \end{cases}$
 $\rightarrow D_f = (-\infty, -1) \cup [\frac{1}{2}, \infty)$
 $\rightarrow D_f = \mathbb{R} - \{-\frac{1}{2}, -1, 2, \frac{5}{2}\}$

الف) $x^2 - x^2 = 0 \rightarrow x^2 = x^2 \rightarrow x = -2$
 $x^2 - x^2 = 0 \rightarrow x^2 = x^2 \rightarrow x = 2, 5$
 $\rightarrow D_f = \mathbb{R} - (-2, 2, 5)$

ب) $x-1 \geq 0 \rightarrow x \geq 1$
 $\sqrt{y+1} = x - \sqrt{x-1} \rightarrow \begin{cases} x \geq \sqrt{x-1} \\ x \geq 1 \end{cases} \rightarrow D_f = [1, \infty)$

$x^2 - x - 2 \geq 0 \rightarrow (x-2)(x+1) \geq 0 \rightarrow \frac{-1}{x+1} \rightarrow (-\infty, -1) \cup (2, +\infty)$

$x^2 - 1 \geq 0 \rightarrow x^2 \geq 1 \rightarrow \begin{cases} x \geq 1 \\ x \leq -1 \end{cases} \rightarrow \mathbb{R}$

$\rightarrow D_f = (-\infty, -1) \cup (2, +\infty)$

عبارت $\frac{1}{x}$ مثبت است $\Rightarrow \Delta \geq 0$
 $x^2 - 2ax - 1 = 0$
 $-2a = 1 \rightarrow a = -\frac{1}{2}$

$\rightarrow -x^2 - \frac{x}{2} + 1 = 0 \rightarrow -2x^2 - x + 2 = 0$
 $x = \frac{1 \pm \sqrt{1+8}}{-2} \rightarrow \begin{cases} x = \frac{3}{2} = b \\ x = -2 \end{cases} \Rightarrow a+b = -\frac{1}{2} + \frac{3}{2} = 1$

$f(x) - x \geq 0 \rightarrow f(x) \geq x$
 $\rightarrow x^2 - 2 \geq x \rightarrow x^2 \geq x+2 \rightarrow x \geq 1$
 $\rightarrow x^2 + 3 \geq x \rightarrow x \geq -3$
 $\rightarrow D_g = [-3, +\infty)$

$$\sin \frac{\pi}{\sqrt{e}} + \cos \sqrt{e} \pi = \sin 20^\circ + \cos 0^\circ = \frac{\sqrt{e}}{\sqrt{e}} + 1 = \frac{e + \sqrt{e}}{\sqrt{e}}$$

$$\rightarrow \frac{e(1 + \sqrt{e})^{\sqrt{e}}}{\sqrt{e}} - \frac{e(1 + \sqrt{e})}{\sqrt{e}} = \frac{e + e + e\sqrt{e} - 1 - e\sqrt{e}}{\sqrt{e}} = \frac{-1}{\sqrt{e}}$$

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$$f(f(a)) = \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} \cdot \frac{\frac{a+1+a-1}{a-1}}{\frac{a+1-a+1}{a-1}} = \frac{2a}{1} = 2a \rightarrow$$

برای هر مقدار از a در دامنه باشد تابع f یک به یک و پاره‌ای است

$$\frac{a+1}{a-1} \neq 1 \rightarrow a+1 \neq a-1 \rightarrow 1 \neq -1$$

پس $a \neq 1$ و $1 \neq -1$ همیشه برقرار است

$$\rightarrow D_f = \mathbb{R} - \{1\}$$

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$$x^2 - 4x = -4$$

$$x^2 - 4x + 4 = 0 \rightarrow (x-2)^2 = 0 \rightarrow x = 2$$

$$\Rightarrow \sqrt{x^2 - 1} = 2 \rightarrow x^2 - 1 = 4 \rightarrow x^2 = 5 \rightarrow x = \pm \sqrt{5} = a$$

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$$\xrightarrow{x=1} f(1) = (1)^2 - \omega(1) + 2 = 1 - \omega + 2 = 2$$

$$\xrightarrow{x=-2} f(-2) = (-2)^2 - \omega(-2) + 2 = 4 + 2\omega + 2 = 7\omega$$

$$\rightarrow 7\omega \times 2 = 14\omega$$

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$$e \cos^2 x + \omega \cos x = e \omega \quad \cos x = t \rightarrow et^2 + \omega t - e\omega = 0$$

روش اول

$$t^2 + \omega t - 1 = 0 \rightarrow (t+1)(t-1) = 0$$

$$t = -1 \rightarrow -\frac{1}{e} = \cos x \quad -1 < \cos x < 1$$

$$t = 1 \rightarrow \frac{1}{e} = \cos x$$

$$\rightarrow \cos x = \frac{1}{e}$$

$$x = 20^\circ \rightarrow f(e, \omega) = (\tan 20^\circ)^2 = (\frac{1}{\sqrt{e}})^2 = \frac{1}{e}$$

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