

$$f(x) = \begin{cases} (a+1)(x+2) & ; x > 1 \\ 3a+2x & ; x \leq 1 \end{cases}$$

$$\begin{aligned} 2f(0) &= f(-2) + a \\ 2(a+1)(0+2) &= 3a + 2(-2) + a \\ 4(a+1) &= 3a - 4 \Rightarrow 4a + 4 = 3a - 4 \Rightarrow a = -8 \end{aligned}$$

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$$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + 2 \quad f(\sqrt{2}-\sqrt{3}) + f(\sqrt{2}+\sqrt{3}) = \sqrt{\sqrt{2}-\sqrt{3}} + \frac{1}{\sqrt{\sqrt{2}-\sqrt{3}}} + \sqrt{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{\sqrt{2}+\sqrt{3}}} + 4$$

$$= 2(\sqrt{\sqrt{2}-\sqrt{3}}) + 2(\sqrt{\sqrt{2}+\sqrt{3}}) + 4 = 2(\sqrt{\sqrt{2}-\sqrt{3}} + \sqrt{\sqrt{2}+\sqrt{3}}) + 4$$

$$2(\sqrt{\sqrt{2}-\sqrt{3}} + \sqrt{\sqrt{2}+\sqrt{3}} + 2) \Rightarrow \boxed{\sqrt{4} + 4}$$

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$$\begin{aligned} 2f(x) - 3f(-x) &= 5x^2 - x \\ 2f(-x) - 3f(x) &= 5x^2 - (-x) = 5x^2 + x \end{aligned} \Rightarrow \begin{cases} 2f(x) - 3f(-x) = 5x^2 - x \\ 2f(-x) - 3f(x) = 5x^2 + x \end{cases} \Rightarrow \begin{cases} 5f(x) - 5f(-x) = -2x \\ f(x) - f(-x) = -\frac{2x}{5} \end{cases}$$

$$\Rightarrow -5f(x) = 10x^2 + x \Rightarrow f(x) = -\frac{2x^2 - x}{5}$$

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$$(x+2)f(x) - 3xf(x+2) = 5x^2 - mx + 3m - 1 \quad f(0) = ?$$

$$\begin{aligned} x=0 \Rightarrow 2f(0) &= 3m - 1 \\ x=-2 \Rightarrow 4f(0) &= 14 + 2m + 3m - 1 = 5m + 13 \end{aligned} \Rightarrow \begin{cases} 2f(0) = 3m - 1 \\ 4f(0) = 5m + 13 \end{cases} \Rightarrow \begin{cases} f(0) = \frac{3m-1}{2} \\ 11m - 4 = 10m + 13 \end{cases}$$

$$f(0) = \frac{3m-1}{2} = \frac{11(-9)-1}{2} = \frac{-100}{2} = -50$$

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$$f(x) + f\left(-\frac{1}{x}\right) = \frac{3x^2 - 12x + 3}{x} \quad f(-1) = ?$$

$$f(x) = ax + b$$

$$ax + b + \frac{a}{x} + b = \frac{3x^2 - 12x + 3}{x} \Rightarrow \begin{cases} a = 3 \\ b = -12 \end{cases}$$

$$f(x) = 3x - 9 \Rightarrow f(-1) = -3 - 9 = -12$$

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$$\begin{cases} f(x) = (x-1)\sqrt{1-x} & D_f = (-\infty, 1] \\ g(x) = -\sqrt{(1-x)^3} & D_g = (-\infty, 1] \end{cases}$$
 $(1-x)^3 \geq 0 \Rightarrow 1-x \geq 0 \Rightarrow x \leq 1$

$\Rightarrow f=g \Rightarrow \sqrt{(x-1)\sqrt{1-x}} = -\sqrt{(1-x)^3}$

$$\begin{cases} f(x) = x\sqrt{x} & D_f = [0, \infty) \\ g(x) = \sqrt{x^3} & D_g = [0, \infty) \end{cases}$$
 $x^3 \geq 0 \Rightarrow x \geq 0$

$\Rightarrow f=g \Rightarrow x\sqrt{x} = \sqrt{x^3}$

$$\begin{cases} f(x) = [\sqrt{x} - \lfloor \sqrt{x} \rfloor] & x \geq 0, D_f = [0, \infty) \\ g(x) = \sqrt{x - |x|} & x - |x| \geq 0, D_g = [0, \infty) \end{cases}$$

$f(x) = [\sqrt{x} - \lfloor \sqrt{x} \rfloor] \Rightarrow \sqrt{x} - \lfloor \sqrt{x} \rfloor \geq 0 \Rightarrow R_f = R_g = 0$

$\Rightarrow f=g$

$f = \{(1,2), (2,3), (3,1), (0,0)\}$
 $g = \{(-1,1), (1,1), (2,3), (3,8), (0,2), (\sqrt{2},1)\}$

الف) $\left(\frac{f+g}{f-g}\right)(1) = \frac{2+1}{2-1} = 3$

ب) $\left(\frac{f+g}{f-g}\right)(2) = \frac{3+3}{3-3} = \frac{6}{0} \Rightarrow \text{غير معرف}$

ج) $f(0)g(-1) = 0 \times 1 = 0$

$f(x) = \{(2,1), (-1,4), (0,8), (-2,2), (3,-1)\}$

الف) $g(x) = f(3x-1) = \{(1,1), (0,4), (\frac{1}{3},8), (\frac{-1}{3},2), (\frac{4}{3},-1)\}$

$3x-1=2 \Rightarrow x=1$
 $3x-1=1 \Rightarrow x=2/3$
 $3x-1=0 \Rightarrow x=1/3$
 $3x-1=-1 \Rightarrow x=0$
 $3x-1=3 \Rightarrow x=4/3$

ب) $h(x) = f(|x|) = \{(2,1), (0,8), (3,-1), (-3,-1), (-2,1)\}$

$|x| \neq -1$
 $|x| \neq -2$
 $|x| = 2 \Rightarrow x = \pm 2$
 $|x| = 0 \Rightarrow x = 0$
 $|x| = 3 \Rightarrow x = \pm 3$