

$$2f(a) = f(-2) + a = 2\left(\frac{1}{a+1}\right)(a+1) = 2a - 4 + a \quad (1)$$

$$\rightarrow 14a + 14 = 3a - 4 \rightarrow 11a = -18 \rightarrow a = -18/11$$

$$f(x - \sqrt{x}) = \sqrt{x - \sqrt{x}} + \frac{1}{\sqrt{x - \sqrt{x}}} + 2 = \sqrt{x - \sqrt{x}} + \frac{\sqrt{x + \sqrt{x}}}{\sqrt{(x - \sqrt{x})(x + \sqrt{x})}} + 2 \quad (2)$$

$$f(x + \sqrt{x}) = \sqrt{x + \sqrt{x}} + \frac{1}{\sqrt{x + \sqrt{x}}} + 2 = \sqrt{x + \sqrt{x}} + \frac{\sqrt{x - \sqrt{x}}}{\sqrt{(x + \sqrt{x})(x - \sqrt{x})}} + 2$$

$$\rightarrow f(x - \sqrt{x}) + f(x + \sqrt{x}) = 2\sqrt{x - \sqrt{x}} + 2\sqrt{x + \sqrt{x}} + 4 = 2(\sqrt{x - \sqrt{x}} + \sqrt{x + \sqrt{x}} + 2)$$

$$\Rightarrow (\sqrt{x - \sqrt{x}} + \sqrt{x + \sqrt{x}})^2 = x - \sqrt{x} + x + \sqrt{x} + 2\sqrt{x - \sqrt{x}}\sqrt{x + \sqrt{x}} = 4 \rightarrow \sqrt{x - \sqrt{x}} + \sqrt{x + \sqrt{x}} = \sqrt{4}$$

$$\rightarrow f(x - \sqrt{x}) + f(x + \sqrt{x}) = 2(\sqrt{4} + 2) = 2\sqrt{4} + 4$$

$$n = n \rightarrow 2f(n) - 2f(-n) = 4n^2 - n \rightarrow 4f(n) - 4f(-n) = 4n^2 - n \quad (3)$$

$$n = -n \rightarrow -2f(n) + 2f(-n) = 4n^2 + n \rightarrow -4f(n) + 4f(-n) = 4n^2 + n$$

$$\rightarrow -4f(n) = 4n^2 - n \rightarrow f(-n) = -n^2 + \frac{1}{4}n$$

$$\rightarrow 2f(n) - 2(-n^2 + \frac{1}{4}n) = 4n^2 - n \rightarrow 2f(n) + 2n^2 - \frac{1}{2}n = 4n^2 - n$$

$$\rightarrow 2f(n) = 2n^2 - \frac{1}{2}n \rightarrow f(n) = -n^2 - \frac{1}{4}n$$

$$(n+2)f(n) - 2nf(n+2) = 4n^2 - mn + 2n - 1 \quad (4)$$

$$\rightarrow n = -2 \rightarrow 4f(0) = 10 + 2m \rightarrow 4f(0) = 10 + 2m$$

$$\rightarrow n = 0 \rightarrow 2f(0) = 2m - 1 \xrightarrow{\times 2} 4f(0) = 4m - 2 \xrightarrow{+} 10 - 4m = 0 \rightarrow m = \frac{5}{2}$$

$$\rightarrow 4f(0) = 10 + \frac{5}{2} = 10 + 2.5 = 12.5 \rightarrow f(0) = 3.125$$

$$f(n) + f\left(\frac{1}{n}\right) = \frac{3n^2 - 12n + 5}{n}$$

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$$f(n) = an + b \rightarrow f(n) + f\left(\frac{1}{n}\right) = an + \frac{1}{n}a + b \xrightarrow{\times n} n(f(n) + f\left(\frac{1}{n}\right))$$

$$f\left(\frac{1}{n}\right) = \frac{1}{n}a + b$$

$$= an^2 + b + a$$

$$\rightarrow f(n) + f\left(\frac{1}{n}\right) = \frac{an^2 + b + a}{n} = \frac{3n^2 - 12n + 5}{n}$$

$$\rightarrow a = 3 \quad b = -4 \rightarrow f(n) = 3n - 4 \rightarrow f(-1) = -3 - 4 = -7$$

$$\begin{cases} f(n) = (n-1)\sqrt{1-n} \rightarrow 1-n \geq 0 \rightarrow n \leq 1 \rightarrow D_f = (-\infty, 1] \end{cases}$$

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$$\begin{cases} g(n) = -\sqrt{(1-n)^5} \rightarrow (1-n)^5 \geq 0 \rightarrow 1-n \geq 0 \rightarrow n \leq 1 \rightarrow D_g = (-\infty, 1] \end{cases}$$

$$\rightarrow \text{برابر} \rightarrow -\sqrt{(1-n)^5(1-n)} = -(1-n)\sqrt{1-n} = (n-1)\sqrt{1-n}$$

$$\begin{cases} f(n) = n\sqrt{n} \rightarrow n \geq 0 \rightarrow D_f = [0, +\infty) \end{cases}$$

$$\begin{cases} g(n) = \sqrt{n^3} \rightarrow n^3 \geq 0 \rightarrow D_g = [0, +\infty) \end{cases} \rightarrow \text{برابر}$$

(7)

$$f(n) = \left[\sqrt{n} - \lfloor \sqrt{n} \rfloor \right] \rightarrow n \geq 0 \rightarrow D_f = [0, +\infty)$$

(8)

$$g(n) = \sqrt{n - |n|} \rightarrow \begin{matrix} + \rightarrow \checkmark \\ - \rightarrow \times \\ 0 \rightarrow \checkmark \end{matrix} \rightarrow D_g = [0, +\infty)$$

$$\rightarrow f(n) = \left[\sqrt{n} - \overset{\text{برابر}}{\lfloor \sqrt{n} \rfloor} \right] = \left[\text{شماره} \right] = 0$$

$$\rightarrow g(n) = \sqrt{n - |n|} \rightarrow \begin{cases} n < 0 \rightarrow 0 \\ n \geq 0 \rightarrow \sqrt{n - n} = 0 \end{cases} \rightarrow \text{برای هر عددی برابر 0 است} \rightarrow \text{برابر}$$

$$f = \{(1, 2)(2, 3)(3, 1)(0, 0)\}$$

$$g = \{(1, 1)(2, 2)(3, 0)(0, 2)(-1, 1)(\sqrt{2}, 1)\}$$

$$f+g = \{(1, 3)(2, 5)(3, 4)(0, 2)\} \quad f-g = \{(1, 1)(2, 0)(3, -4)(0, -2)\}$$

$$\text{ا) } \left(\frac{f+g}{f-g}\right)_n = \{(1, 3)(2, \frac{5}{-4})(3, -\frac{4}{-2})(0, -1)\} \rightarrow \left(\frac{f+g}{f-g}\right)(1) = \boxed{5}$$

$$\text{ب) } \left(\frac{f+g}{f-g}\right)_n = \{ \quad \quad \quad \} \rightarrow \left(\frac{f+g}{f-g}\right)(2) = \boxed{\frac{5}{4} \text{ غير صحيح}}$$

$$\text{ج) } f(0) g(-1) = 0 \times 1 = 0$$

$$f(n) = \{(2, 1)(-1, 4)(0, 0)(-2, 2)(3, -1)\}$$

$$\text{ا) } g(n) = f(r_{n-1}) = \{(1, 1)(0, 4)(\frac{1}{r}, 0)(-\frac{1}{r}, 2)(\frac{4}{r}, -1)\}$$

$$r_{n-1} = 2 \rightarrow r_n = 4 \rightarrow n = 1 \quad r_{n-1} = -1 \rightarrow r_n = 0 \rightarrow n = 0 \quad r_{n-1} = 0 \rightarrow r_n = 1 \rightarrow n = \frac{1}{r}$$

$$r_{n-1} = -2 \rightarrow r_n = -1 \rightarrow n = -\frac{1}{r} \quad r_{n-1} = 3 \rightarrow r_n = 4 \rightarrow n = \frac{4}{r}$$

$$\text{ب) } h(n) = f(|n|) = \{(2, 1)(-2, 1)(0, 0)(3, -1)(-3, 1)\}$$