

$$f(x) = \begin{cases} (a+1)(x+r) & x > 1 \\ ra + rx & x \leq 1 \end{cases}$$

$$\begin{aligned} rf(0) &= f(-r) + a \\ r(a+1)r &= ra - f + a \\ 1fa + 1f &= ra - f + a \\ 10a &= -1A \\ \underline{a = -1A} \end{aligned}$$

a = ?

(1)

$$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + r$$

$$\begin{aligned} f(r-\sqrt{r}) + f(r+\sqrt{r}) &\rightarrow \frac{\sqrt{(r-\sqrt{r})(r+\sqrt{r})}}{\frac{\sqrt{r-\sqrt{r}}}{a} \cdot \frac{\sqrt{r+\sqrt{r}}}{\frac{1}{a}}} = 1 \Rightarrow a + \frac{1}{a} + r + \frac{1}{a} + a + r = \Rightarrow \\ &\quad r(a + \frac{1}{a} + r) \Rightarrow r(\sqrt{r-\sqrt{r}} + \sqrt{r+\sqrt{r}} + r) \\ &\quad \checkmark \hookrightarrow (a + \frac{1}{a})^r \Rightarrow (\sqrt{r-\sqrt{r}} + \sqrt{r+\sqrt{r}})^r \Rightarrow r + r \cdot r \rightarrow \sqrt{r} \end{aligned}$$

(2)

$$\begin{aligned} rf(x) - rf(-x) &= rx^r - x \\ \{ \begin{aligned} rf(x) &= rf(-x) = 1x^r - x \\ rf(-x) &= 9f(x) = 11x^r + rx \end{aligned} \} \rightarrow \begin{aligned} -of(x) &= rx^r + x \\ f(x) &= \frac{rx^r + x}{-5} \Rightarrow f(x) = -\frac{rx^r - x}{5} \end{aligned} \end{aligned}$$

(3)

$$\begin{aligned} (x+r)f(x) - rx f(x+r) &= rx^r - mx + rm - 1 \xrightarrow{x=0} \begin{cases} rf(0) = rm - 1 \xrightarrow{x=r} rf(r) = 9m - r \\ rf(0) = 1r + 8m - 1 \end{cases} \quad (4) \\ f(0) = ? \rightarrow \frac{rf(0) - 1}{r} \Rightarrow \frac{rv - 1}{r} = \boxed{\frac{ra}{f}} \\ \Rightarrow 1r + 8m - 1 = 9m - r \\ 1A = fm \\ \boxed{f(0) = \frac{1A}{f} = m} \end{aligned}$$

$$f(x) + f\left(\frac{1}{x}\right) = \frac{rx^r - 1rx + r}{x} \xrightarrow{x=1} rf(1) \Rightarrow \frac{r+1r+r}{-1} \Rightarrow -1A$$

(5)

$$\begin{aligned} f(-1) = ? \quad ax + b \cdot \frac{a}{x} + b &= \frac{rx^r - 1rx + r}{x} \quad \boxed{f(-1) = -9} \\ \frac{rx^r - 1rx + r}{x} = \frac{ax^r + bx + a + bn}{x} \end{aligned}$$

$$\begin{aligned} \left\{ \begin{aligned} f(x) &= (x-1)\sqrt{1-x} \rightarrow \\ g(x) &= -\sqrt{(1-x)^3} \rightarrow \begin{cases} (1-x)^3 \gg 0 \\ (1-x) \gg 0 \rightarrow 1 \gg x \end{cases} \end{aligned} \right. \end{aligned}$$

(6)

$$\begin{aligned} \left\{ \begin{aligned} f(x) &= x\sqrt{x} \rightarrow x \gg 0 \rightarrow Df = (0, +\infty) \\ g(x) &= \sqrt{x^3} \rightarrow x^3 \gg 0 \rightarrow x \gg 0 \rightarrow Dg = (0, +\infty) \end{aligned} \right. \end{aligned}$$

(7)

$$f(x) = [\sqrt{x} - \lfloor \sqrt{x} \rfloor] \Rightarrow x \geq 0 \quad Df = [0, +\infty)$$

①

$$g(x) = \sqrt{x - |x|} \Rightarrow x - |x| \geq 0 \rightarrow x \geq |x| \rightarrow Dg = [0, +\infty)$$

$$g(x) = 0$$

$$f(x) \xrightarrow{x \rightarrow 1} [\sqrt{x} - \lfloor \sqrt{x} \rfloor] \rightarrow (\sqrt{1} - 1) = 0 \quad \text{ب } 0 < \sqrt{x} - \lfloor \sqrt{x} \rfloor < 1$$

②

$$f = \{(1, 2), (2, 3), (3, 1), (0, 0)\}$$

$$g = \{(-1, 1), (1, 1), (2, 3), (3, 0), (0, 2), (\sqrt{2}, 1)\}$$

$$\text{ا) } \left(\frac{f+g}{f-g} \right)(1) \Rightarrow \frac{2+1}{2-1} = 3$$

$$\text{ب) } \left(\frac{f+g}{f-g} \right)(2) \Rightarrow \frac{3+3}{3-3} \Rightarrow \text{undefined}$$

$$\text{ج) } f(0)g(-1) = 0 \times 1 = 0$$

③

$$f(x) = \{(2, 1), (-1, 4), (0, 0), (-2, 2), (3, -1)\}$$

$$\text{ا) } g(x) = f(3x-1) \rightarrow g(x) = \{(1, 1), (0, 4), (\frac{1}{3}, 0), (-\frac{1}{3}, 2), (\frac{2}{3}, -1)\}$$

$$2 = 3x - 1$$

$$x = 1$$

$$-1 = 3x - 1$$

$$x = 0$$

$$0 = 3x - 1$$

$$x = \frac{1}{3}$$

$$-2 = 3x - 1$$

$$x = -\frac{1}{3}$$

$$3 = 3x - 1$$

$$x = \frac{4}{3}$$

$$\text{ب) } h(x) = f(|x|)$$

$$h(x) = \{(-2, 1), (-1, 4), (0, 0), (2, 1), (3, -1)\}$$

$$|x| = 2$$

$$x = 2, -2$$

$$|x| = 1$$

$$|x| = 0$$

$$x = 0$$

$$|x| = -2$$

$$|x| = 3$$

$$x = 3, -3$$