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$$f(a) = f(a+1) \Rightarrow 1/a + 1/a$$

$$f(-1) = 1/a - 1$$

$$1/a + 1/a = 1/a - 1 \rightarrow 1/a = -1 \rightarrow a = -1$$

$$x = 1 + \sqrt{2} \rightarrow \sqrt{x} = \sqrt{1 + \sqrt{2}} \Rightarrow \sqrt{\frac{2(1 + \sqrt{2})}{2}} \rightarrow \sqrt{\frac{1 + \sqrt{2}}{2}} = \frac{\sqrt{1 + \sqrt{2}}}{\sqrt{2}}$$

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$$x = 1 - \sqrt{2} \rightarrow \sqrt{x} = \sqrt{1 - \sqrt{2}} = \frac{\sqrt{1 - \sqrt{2}}}{\sqrt{2}}$$

$$f(1 + \sqrt{2}) = \frac{\sqrt{1 + \sqrt{2}}}{\sqrt{2}} + \frac{\sqrt{2}}{\sqrt{1 + \sqrt{2}}} + 1 = \frac{\sqrt{1 + \sqrt{2}}}{\sqrt{2}} + \frac{\sqrt{2}(\sqrt{1 + \sqrt{2}})}{1 + \sqrt{2}} + 1 = \frac{\sqrt{1 + \sqrt{2}} + \sqrt{2} + \sqrt{1 + \sqrt{2}}}{\sqrt{2}} + 1 = \frac{2\sqrt{1 + \sqrt{2}} + \sqrt{2}}{\sqrt{2}} + 1 = \sqrt{2} + 1$$

①

$$f(1 - \sqrt{2}) = \frac{\sqrt{1 - \sqrt{2}}}{\sqrt{2}} + \frac{\sqrt{2}}{\sqrt{1 - \sqrt{2}}} + 1 = \sqrt{2} + 1$$

$$\text{① و ② جمع} = 2\sqrt{2} + 2$$

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$$f(x) - f(-x) = x^2 - x^2 \rightarrow f(x) - f(-x) = 1/x^2 - 1/x^2 \rightarrow -\Delta f(x) = 1/x^2 + x \rightarrow f(x) = -1/x^2 - \frac{1}{2}x$$

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$$x = 0: f(0) = 1/m - 1 \rightarrow f(0) = \frac{m-1}{m}$$

$$x = -1: f(-1) = 1/m + 1/m - 1 \rightarrow f(-1) = \frac{1+m}{m}$$

$$\frac{m-1}{m} = \frac{1+m}{m} \rightarrow m-1 = 1+m \rightarrow m = 1 \rightarrow m = \frac{9}{2}$$

$$f(0) = \frac{m-1}{m} = \frac{\frac{9}{2}-1}{\frac{9}{2}} = \frac{7}{9}$$

$$f(x) + f\left(\frac{1}{x}\right) = 1/x - 1/x + \frac{1}{x}$$

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$$ax + b + \frac{a}{x} + b = 1/x + \frac{1}{x} - 1/x \rightarrow a = 1, b = -1/2$$

$$f(x) = 1/x - 1/2 \rightarrow f(-1) = 1/(-1) - 1/2 = -3/2$$

$$f(x) = (x-1)\sqrt{1-x} \rightarrow D_f: 1-x \geq 0 \rightarrow x \leq 1$$

$$g(x) = -\sqrt{(1-x)^3} = -\sqrt{(1-x)^2(1-x)} = -(1-x)\sqrt{1-x} = (x-1)\sqrt{1-x} \rightarrow D_g = (-\infty, 1]$$

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دامنه و ضابطه یکی است پس تابع برابری

$$\begin{cases} f(x) = x\sqrt{x} \rightarrow x \geq 0 \\ g(x) = \sqrt{x^3} \rightarrow x^3 \geq 0 \rightarrow x \geq 0 \end{cases}$$

دامنه ها برابر

ضابطه ها هم یکی هستند $x'x x^{\frac{1}{2}} = x^{\frac{3}{2}}$

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$$\begin{cases} f(x) = [\sqrt{x} - \lfloor \sqrt{x} \rfloor] \\ g(x) = \sqrt{x - \lfloor x \rfloor} \end{cases}$$

تابع‌های f و g برای $x \geq 0$ صفر است
اما فقط برای مقبوضه‌های کامل
است پس دافعه‌ی تابع‌ها با هم برابر x نیستند

الف) $\frac{f(1) + g(1)}{f(1) - g(1)} = \frac{2+1}{2-1} = 3$

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ب) $\frac{f(2) + g(2)}{f(2) - g(2)} = \frac{2+2}{2-2} \rightarrow \text{ت.ن.}$

ج) $0 \times 1 = 0$

الف) $g(x) = f(3x-1)$

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$$3x-1 \in \mathbb{Z}^+ \rightarrow x \in \mathbb{Z}^+$$

$$3x-1 \in \mathbb{Z}^- \rightarrow x \in \mathbb{Z}^-$$

$$3x-1 \in \mathbb{Q} \rightarrow x = \frac{1}{3}$$

$$3x-1 \in \mathbb{R} \rightarrow x \in \mathbb{R}$$

$$3x-1 \in \mathbb{C} \rightarrow x \in \mathbb{C}$$

ب) $h(x) = f(|x|)$

$$x \in \mathbb{Z}, -2, 0, 2$$