

۲. آزمون های ورنیه

مادران محترم! تخمین ۴ مورد

$$f(n) = \begin{cases} (n+1)(n+c) & n \geq 1 \\ c n^2 + n & n = 1 \end{cases} \Rightarrow f(1) = f(-1) + c \Rightarrow f((n+1)(1)) = c n + (c+c) n$$

$$\Rightarrow 15n + 15 = 4n - 2 \Rightarrow 10n = -17 \Rightarrow n = \frac{-17}{10}$$

①

$$f(n) = \sqrt{n} + \frac{1}{\sqrt{n}} + 2 \quad f(\sqrt{2}-\sqrt{c}) + f(\sqrt{2}+\sqrt{c}) \Rightarrow \sqrt{2-\sqrt{c}} + \frac{1}{\sqrt{2-\sqrt{c}}} + \sqrt{2+\sqrt{c}} + \frac{1}{\sqrt{2+\sqrt{c}}} + 2$$

$$\Rightarrow \frac{(\sqrt{2-\sqrt{c}})^2 + 1}{\sqrt{2-\sqrt{c}}} + \frac{(\sqrt{2+\sqrt{c}})^2 + 1}{\sqrt{2+\sqrt{c}}} + 2 \Rightarrow \frac{(2-\sqrt{c})(\sqrt{2+\sqrt{c}}) + \sqrt{2+\sqrt{c}} + (2+\sqrt{c})(\sqrt{2-\sqrt{c}}) + \sqrt{2-\sqrt{c}}}{\sqrt{2-\sqrt{c}} \sqrt{2+\sqrt{c}}} + 2$$

در اینجا فرض کنید  $\sqrt{2-\sqrt{c}} = a$  و  $\sqrt{2+\sqrt{c}} = b$

②

$$\left. \begin{aligned} cf(n) - cf(n) &= En^c - n \\ f(n) &? \end{aligned} \right\} \Rightarrow (cf(n) - cf(n) = En^c - n) \times 2$$

$$\Rightarrow cf(n) - cf(n) = 12n^c - 2n \Rightarrow -2f(n) = 12n^c + 2n \Rightarrow f(n) = \frac{12n^c + 2n}{-2}$$

$$cf(n) - 9f(n) = 12n^c + 2n \Rightarrow -8f(n) = 12n^c + 2n \Rightarrow f(n) = \frac{12n^c + 2n}{-8}$$

③

$$(n+c)f(n) - cf(n+c) = En^c - mn + 2m - 1 \quad f(0)?$$

$$cf(0) + 0 = cm - 1 \quad cf(0) = 9m - 5$$

$$n=2 \Rightarrow (5+c)f(2) + 4f(0) = 14 + 2m + cm - 1 \quad 4f(0) = 14 + 4m$$

$$cf(0) = \frac{14}{4} - \frac{m}{1} \Rightarrow \frac{14}{4} \Rightarrow f(0) = \frac{14}{4}$$

④

$$f(n) + f\left(\frac{1}{n}\right) = \frac{12n^c - 12n + 2}{n} \Rightarrow f(-1)?$$

$$\Rightarrow cf(-1) = \frac{12 + 12 + 2}{-1} \Rightarrow cf(-1) = -14$$

اینجا فرض کنید  $n = -1$

⑤

$$f(x) = (x-1)\sqrt{1-x} \quad g(x) = -\sqrt{(1-x)^3} \quad D_f \Rightarrow 1-x \geq 0 \quad x \leq 1$$

$$D_g: 1-x \geq 0 \quad x \leq 1 \quad \Rightarrow D_f = D_g$$

$$g(x) = -\sqrt{(1-x)^3} \Rightarrow -\sqrt{(1-x)^2(1-x)} \xrightarrow{\text{cancel}} \cancel{-(1-x)} \sqrt{(1-x)} = (1-x)\sqrt{(1-x)}$$

$$\text{نکته: } (x-1)\sqrt{(1-x)} \Rightarrow g(x) = f(x) \quad \text{✓}$$

$$f(x) = x\sqrt{x} \quad g(x) = \sqrt{x^3} \quad D_f: x \geq 0 \quad D_g: x \geq 0 \quad D_f = D_g$$

$$g(x) = \sqrt{x^3} = \sqrt{x^2 \cdot x} = x\sqrt{x} \Rightarrow g(x) = f(x) \quad \text{✓}$$

$$f(x) = [\sqrt{x} - \lfloor \sqrt{x} \rfloor] \quad g(x) = \sqrt{x - \lfloor x \rfloor} \quad D_f: x \geq 0 \quad D_g: x \geq 0$$

$$\Rightarrow D_f = D_g \quad f(x) \Rightarrow \sqrt{x} - a \quad a = \lfloor \sqrt{x} \rfloor \quad a - \lfloor a \rfloor = \begin{cases} \text{if } a \in \mathbb{Z} \dots \\ \text{if } a \notin \mathbb{Z} \Rightarrow a < 1 \end{cases}$$

$$\begin{cases} \text{if } a \in \mathbb{Z} \Rightarrow [a - \lfloor a \rfloor] = 0 \\ \text{if } a \notin \mathbb{Z} \Rightarrow [a - \lfloor a \rfloor] = 0 \end{cases} \quad f(x) = g(x) = \sqrt{x - \lfloor x \rfloor} \Rightarrow x \geq 0 \Rightarrow \sqrt{0} = 0$$

$$\Rightarrow g(x) = f(x) \quad \text{✓}$$

$$f(x) = g(1) = 0 \times 1 = 0 \quad \left( \frac{f+g}{f-g} \right)(x) = \frac{x+1}{x-1} \Rightarrow \frac{f(1)+g(1)}{f(1)-g(1)} = \frac{0}{0} \quad \text{✓}$$

$$h(x) = f(x) \quad g(x) = f(x-1)$$

$$h(x) = \left\{ \left( \frac{1}{e} + 1 \right) \left( -\frac{1}{e} \right), \left( \frac{1}{e} + 1 \right) \left( \frac{1}{e} - 1 \right), \left( \frac{1}{e} - 1 \right) \right\} \quad g(x) = \left\{ \left( \frac{1}{e} + 1 \right) \left( -\frac{1}{e} + 1 \right), \left( \frac{1}{e} - 1 \right) \right\}$$

ساده بنویس



$$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} \Rightarrow f(x) = \sqrt{x} + \frac{1}{\sqrt{x}}$$

$$\Rightarrow \sqrt{x} + \frac{1}{\sqrt{x}} + \sqrt{x} + \frac{1}{\sqrt{x}} = 2\sqrt{x} + \frac{2}{\sqrt{x}}$$

$$\frac{(\sqrt{x})^2 + 1}{\sqrt{x}} = \frac{x + 1}{\sqrt{x}}$$

$$\Rightarrow \sqrt{x} + \frac{1}{\sqrt{x}} = \frac{x + 1}{\sqrt{x}}$$

$$\frac{1}{\sqrt{x}} = \frac{1}{\sqrt{x}}$$

$$\sqrt{x} + \frac{1}{\sqrt{x}} = \frac{x + 1}{\sqrt{x}}$$

$$\Rightarrow \sqrt{x} + \frac{1}{\sqrt{x}} = \frac{x + 1}{\sqrt{x}}$$

$$\frac{1}{\sqrt{x}} = \frac{1}{\sqrt{x}}$$

$$\frac{1}{\sqrt{x}} = \frac{1}{\sqrt{x}}$$

$$\frac{1}{\sqrt{x}} = \frac{1}{\sqrt{x}}$$

$$\frac{1}{\sqrt{x}} = \frac{1}{\sqrt{x}}$$