

①

$$f(x) = \begin{cases} (x+1)(x+2) & x > 1 \\ 3x+2x & x \leq 1 \end{cases}$$

$$2f(0) = f(-2) = a$$

$$2(a+1)(1) = 3a-4+a$$

$$14a+14 = 4a-4$$

$$10a = -18$$

$$a = \frac{-18}{10} = -1.8$$

②

$$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + 2 \rightarrow \frac{x+1}{\sqrt{x}} + 2$$

$$\sqrt{2-\sqrt{2}} + \frac{1}{\sqrt{2-\sqrt{2}}} + 2 + \sqrt{2+\sqrt{2}} + \frac{1}{\sqrt{2+\sqrt{2}}} + 2$$

$$2 \left(\underbrace{\sqrt{2-\sqrt{2}} + \sqrt{2+\sqrt{2}}}_A + 2 \right) = 2\sqrt{4+2}$$

$$A^2 = 2 + \sqrt{2} + 2 - \sqrt{2} + 2\sqrt{4-2} = 8$$

③

$$2f(x) - 3f(-x) = 1x^2 - x$$

$$x \rightarrow 2f(x) - 3f(-x) = 1x^2 - x$$

$$x \rightarrow -3f(x) + 2f(-x) = 1x^2 + x$$

$$\begin{cases} 2f(x) - 3f(-x) = 1x^2 - x \\ -3f(x) + 2f(-x) = 1x^2 + x \end{cases}$$

$$-5f(x) = 10x^2 + x$$

$$f(x) = -2x^2 - \frac{x}{5}$$

$$(n+r)f(n) - rn f(n+r) \leq rn^2 - mn + m-1$$

$$\begin{cases} n \leq -r \rightarrow f(n) \leq 19 + 2m + m-1 \rightarrow f(n) \leq 19 + 3m \\ n \leq 0 \rightarrow f(n) \leq m-1 \\ n \geq 0 \rightarrow \begin{cases} 1. f(n) \leq m-1 \\ 2. f(n) \leq 19 + 3m \end{cases} \end{cases}$$

$$f(n) \leq m$$

$$f(n) \leq \frac{m}{n} = \frac{1}{n}$$

$$f(n) + f\left(\frac{1}{n}\right) = \frac{m^2 - 12n + 1}{n}$$

$$\frac{a}{n} + b + \frac{a}{n} + b = \frac{m^2 - 12n + 1}{n}$$

$$f(-1) = a(-1) + b = -a + b = 1$$

$$f(-1) + f(-1) = \frac{m^2 - 12 + 1}{-1}$$

$$2f(-1) = -11$$

$$f(-1) = -\frac{11}{2}$$

$$2b - 2a = -11$$

$$b - a = -\frac{11}{2}$$

$$\begin{cases} f(n) \leq (n-1)\sqrt{1-n} \\ g(n) \leq \sqrt{(1-n)^2} \end{cases}$$

$$1-n \geq 0$$

$$n \leq 1$$

$$(1-n)^2 \geq 0$$

$$n \leq 1$$

$$\begin{aligned} n \leq 0 &\rightarrow -1\sqrt{1} = -1 \\ &\rightarrow -\sqrt{1^2} = -1 \end{aligned}$$

مساوی است

$$\begin{cases} f(n) \leq n\sqrt{n} \\ g(n) \leq \sqrt{n^2} \end{cases}$$

$$n \geq 0$$

$$n^2 \geq 0$$

$$n \geq 0$$

$$n \leq 1 \rightarrow 1\sqrt{1} = 1$$

$$\rightarrow \sqrt{1^2} = 1$$

مساوی است

$$\begin{cases} f(n) \leq [\sqrt{n}] \\ g(n) \leq \sqrt{n-1} \end{cases}$$

$$n \geq 0 \rightarrow A = \sqrt{n}$$

$$[A - [A]]$$

$$A \in \mathbb{Z} \rightarrow 0$$

$$A \notin \mathbb{Z} \rightarrow [] < 1 = 0$$

$$n - 1 \geq 0$$

$$n \geq 1$$

$$n \geq 0$$

$$g(n) \leq 0$$

مساوی است

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$$f = \{ (1,1) (2,3) (3,1) (0,0) \}$$

$$g = \{ (-1,1) (1,1) (2,3) (3,1) (0,0) (\sqrt{1},1) \}$$

الف) $\frac{x+1}{x-1} = \frac{x}{1} = x \quad \text{ب) } \frac{x+3}{x-3} \text{ تعرفت ج) } 0+1 \leq 1$

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$$f(n) = \{ (2,1) (-1,4) (0,5) (-2,2) (3,-1) \}$$

الف) $g(n) = f(3n-1)$

$$g(n) = \{ (1,1) (0,4) (\frac{1}{3},5) (-\frac{1}{3},2) (0,-1) \}$$

$$\begin{aligned} 3n-1 \leq 2 &\rightarrow 3n \leq 3 \rightarrow n \leq 1 \quad (1,1) \\ 3n-1 \leq -1 &\rightarrow 3n \leq 0 \rightarrow n \leq 0 \quad (0,4) \\ 3n-1 \leq 0 &\rightarrow 3n \leq 1 \rightarrow n \leq \frac{1}{3} \quad (\frac{1}{3},5) \\ 3n-1 \leq -2 &\rightarrow 3n \leq -1 \rightarrow n \leq -\frac{1}{3} \quad (-\frac{1}{3},2) \\ 3n-3 \leq 3 &\rightarrow 3n \leq 6 \rightarrow n \leq 2 \quad (0,-1) \end{aligned}$$

ب) $h(n) = f(|n|)$

$$h(n) = \{ (2,1) (-2,1) (0,5) (2,3) (-2,3) \}$$

$$\begin{aligned} |n| \leq 2 &\rightarrow n \leq 2 \\ |n| \leq -1 &\text{ لا } \\ |n| \leq 0 &\rightarrow n \leq 0 \\ |n| \leq -2 &\text{ لا } \\ |n| \leq 3 &\rightarrow n \leq 3 \end{aligned}$$